HOMEWORK #4

DATE ASSIGNED: NOVEMBER 15, 2017; DUE: WEDNESDAY, DECEMBER 6, 2017

Write up carefully problems (1) and (2), and four of the induction proofs from (3) to (8). Make sure to clearly indicate your base case, inductive assumption and your conclusion (you must have a statement which says that you conclude that some statement is true for all values greater than the base case).

(1) Let $L_1 = 1, L_2 = 3$ and $L_{n+1} = L_n + L_{n-1}$ for $n \geq 3$. Prove by induction that
$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$
for $n \geq 1$.

(2) Show that postage stamps of value 5 cents and 9 cents are sufficient to post any letter requiring more than 31 cents in postage.

(3) Let n be a positive integer and $a_i \geq 0$ for $1 \leq i \leq n$. Show that
$$(1+a_1)(1+a_2)\cdots(1+a_n) \geq \frac{2^n}{n+1}(1+a_1+a_2+\cdots+a_n).$$

(4) Show that for $n > 0$,
$$2!4!\cdots(2n)! \geq ((n+1)!)^n \ .$$

(5) Show that for $n \geq 1$,
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1 \ .$$

(6) Let $f_0(x) = \frac{1}{1-x}$, and define $f_{n+1}(x) = x f_n'(x)$. Prove that $f_{n+1}(x) > 0$ for $0 < x < 1$.

(7) Show that for $n \geq 1$,
$$1 + 2\cos x + 2\cos 2x + 2\cos 3x + \cdots + 2\cos nx = sin((2n+1)x/2)/sin(x/2) \ .$$

(8) Show that
$$(\cos x + \cos 2x + \cdots + \cos nx)\sin\left(\frac{x}{2}\right) = \cos\left(\left(\frac{x}{2}\right)(n+1)\right)\sin\left(\frac{nx}{2}\right) \ .$$