ASSIGNMENT 1

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Question: The figure below is supposed to represent a much larger figure made of up triangles where the base of each of the small triangles is of unit length. The figure below is 6 units on a side, but imagine that it is one of a sequence of figures where the n th figure has length n units on a side.

(a) How many triangles can be found in the r^{th} row from the top where $1 \leq r \leq n$?

In the r^{th} row there are $2r - 1$ triangles.

In the first row there is one triangle and in every row there is the same number of triangles as in the previous row plus two more. And in the beginning (for the first row) there is 1. If the r^{th} there are $2r - 1$, and the $r + 1^{st}$ row has 2 more then the $r + 1^{st}$ row has $2r - 1 + 2 = 2r + 1$. Then by the principle of mathematical induction, there are $2r - 1$ triangles in the r^{th} row.

(b) What is the area of the large triangle? What is the area of the large triangle divided by the small triangle?

Heron's formula

`http://www.mathwarehouse.com/geometry/triangles/area/herons-formula-triangle-area.php`

says that if the perimeter of the triangle is $2S$ and the side lengths are A, B, C then the area is equal to $\sqrt{S(S-A)(S-B)(S-C)}$. In our picture the length of the side is n so $A = B = C = n$ and $S = 3n/2$. By this formula the area of the larger triangle is equal to $\sqrt{(3n/2)(3n/2-n)(3n/2-n)(3n/2-n)} = (\sqrt{3}/4)n^2$. This is the area of the larger triangle.

The area of the smaller triangle is proportional because the length of the side is 1 instead of n and so the area of each of the smaller triangles is equal to $\sqrt{3}/4$. The area of the large triangle divided by the smaller triangle is n^2.

(c) Use the general figure to argue that

$$1 + 3 + 5 + \cdots + 2n - 1 = n^2$$

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The number of small triangles is equal to $1+3+5+\cdots+(2n-1)$ and the area occupied those triangles is going to be $(1+3+5+\cdots+(2n-1))\sqrt{3}/4$. The area of the larger triangle is equal to $(\sqrt{3}/4)n^2$. Since the areas of the sums of the smaller triangles and the larger are equal, then we know that

$$(1+3+5+\cdots+(2n-1))\sqrt{3}/4 = (\sqrt{3}/4)n^2$$

and so

$$(1+3+5+\cdots+(2n-1)) = n^2\ .$$

(1) look for simarlities and patterns
(2) guess and check and alter (if it doesn't work)
(2.5) generate data, do examples
(3) check the differences and the second differences

For example: our sequence increases like $1, 4, 9, 16, 25$ and the first differences are $1, 3, 5, 7, 9$ the fact that those increase linearly means that the formula for the sequence itself will have a square.

(4) look for patterns, generalize
(5) rewrite your question in different language
(6) look it up (roughly, this means ask Google)
(7) ask someone who knows the answer
(8) learn more math

Problem: Show that n^2+n+41 is prime for $n \geq 1$

$n=$	n^2+n	n^2+n+41
1	2	43
2	6	47
3	12	53
4	20	61
5	30	71
6	42	83
7	56	97
8	72	113
$\vdots$		
40	$40\cdot 41$	$40\cdot 41+41=41^2$