PRACTICE PROBLEMS

(1) Prove that if an integer n has the form $6q+5$ for some $q \in \mathbb{Z}$, then n also is of the form $3k+2$ for some $k \in \mathbb{Z}$.

(2) Prove or disprove the converse of the statement "if an integer n has the form $6q+5$ for some $q \in \mathbb{Z}$, then n also is of the form $3k+2$ for some $k \in \mathbb{Z}$."

(3) Let x be a real number. Prove or disprove, $x = 2$ if and only if $x^3 - x^2 - x = 2$.

(4)

$$1 \quad 2 \quad 3 \quad \cdots \quad n \quad (n+1) \quad \cdots \quad (2n-2) \quad (2n-1) \quad (2n)$$

Consider the natural numbers from 1 to $2n$. Pair off these numbers as above, 1 and $(2n)$, 2 and $(2n-1)$, 3 and $(2n-2)$, $\ldots$, n and $(n+1)$, and evaluate the products of the pairs, $1 \times (2n)$, $2 \times (2n-1)$, $3 \times (2n-2)$, $\ldots$, $n \times (n+1)$. Prove that for no value of n are two of these n products equal.

(5) Prove by induction that the sum of the cubes of three consecutive positive numbers is divisible by 9.

(6) Prove that since either n is $3r$, $3r+1$ or $3r+2$ for some integer r, then $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.