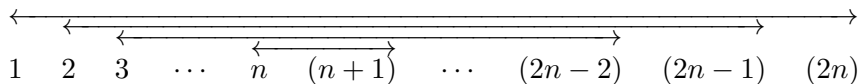


## PRACTICE PROBLEMS

- (1) Prove that if an integer  $n$  has the form  $6q + 5$  for some  $q \in \mathbb{Z}$ , then  $n$  also is of the form  $3k + 2$  for some  $k \in \mathbb{Z}$ .
- (2) Prove or disprove the converse of the statement “if an integer  $n$  has the form  $6q + 5$  for some  $q \in \mathbb{Z}$ , then  $n$  also is of the form  $3k + 2$  for some  $k \in \mathbb{Z}$ .”
- (3) Let  $x$  be a real number. Prove or disprove,  $x = 2$  if and only if  $x^3 - x^2 - x = 2$ .
- (4)



Consider the natural numbers from 1 to  $2n$ . Pair off these numbers as above, 1 and  $(2n)$ , 2 and  $(2n - 1)$ , 3 and  $(2n - 2)$ ,  $\dots$ ,  $n$  and  $(n + 1)$ , and evaluate the products of the pairs,  $1 \times (2n)$ ,  $2 \times (2n - 1)$ ,  $3 \times (2n - 2)$ ,  $\dots$ ,  $n \times (n + 1)$ . Prove that for no value of  $n$  are two of these  $n$  products equal.

- (5) Prove by induction that the sum of the cubes of three consecutive positive numbers is divisible by 9.
- (6) Prove that since either  $n$  is  $3r$ ,  $3r + 1$  or  $3r + 2$  for some integer  $r$ , then  $n^3 + (n + 1)^3 + (n + 2)^3$  is divisible by 9.