1. Show that for $n \geq 1$,
$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} = \frac{2^{n+1} - n - 2}{2^n} .$$

2. Let x, y and a be real numbers. Prove that if $x + y \geq 2a$, then $x \geq a$ or $y \geq a$.

3. Let $f(n) = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)}$.
   (a) Calculate $f(1), f(2), f(3), f(4)$.
   (b) Conjecture a formula for $f(n)$ for $n \geq 1$.
   (c) Prove your formula by induction.

4. Find a and b such that a and b are real and
$$a + bi = \frac{1+i}{1-i} .$$

5. Find all complex values of x such that $x^2 = 1 + i$.

6. Prove that if a is rational and b is irrational then $a + b$ is irrational and ab is irrational.

7. Let $a_0, a_1, a_2, a_3, \ldots$ be a sequence of numbers for $n \geq 0$ defined so that $a_0 = 1$ and $a_n = 4a_{n-1} - n$ for $n \geq 1$. Prove that $a_n = \frac{5 \cdot 4^n + 3n + 4}{9}$.

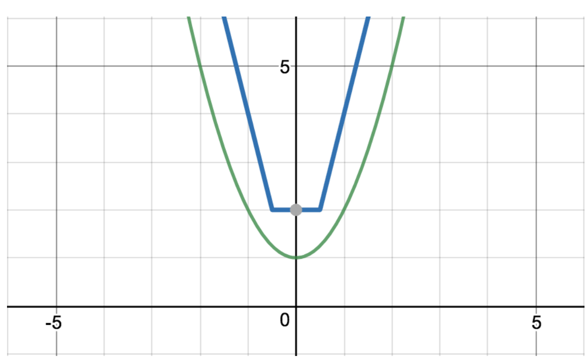
8. Prove that for $n \geq 1$,
$$(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + \cdots + C(n,n-1)xy^{n-1} + C(n,n)y^n$$
   where the numbers $C(n,k)$ are defined for $n \geq 1$, $C(n,0) = C(n,n) = 1$ and $C(n,k) = C(n-1,k-1) + C(n-1,k)$ for $1 \leq k \leq n-1$.

9. Prove or disprove the statement: "If x is a real number s.t. $x^2 + 1 \leq 0$, then $|2x+1| + |2x-1| \leq 4$."

10. Prove or disprove the statement: "If x is a real number s.t. $x^2 + 1 \leq 1$, then $|2x+1| + |2x-1| \leq 4$."

11. Prove or disprove the statement: "If x is a real number s.t. $x^2 + 1 \leq 2$, then $|2x+1| + |2x-1| \leq 4$."

12. Prove or disprove the statement: "If x is a real number s.t. $x^2 + 1 \leq 5$, then $|2x+1| + |2x-1| \leq 4$."



The graphs of $y = x^2 + 1$ and $y = |2x+1| + |2x-1|$.