Hi All! I will have music playing in the background at least until 10am so that people can test their speakers.
Hey, if you hate my music you might want to turn off the volume until then. :)

If you need to play with the controls the menu next to “Start Video” has a option “Video Settings…” and then then there is an “Audio” tab where you can test speakers/microphone

When you come into the meeting your microphone should by default be mute

You may communicate by

1. the chat
2. signals of yes/no/faster/slower/thumbs up/thumbs down/etc.
3. you may unmute your microphone (but remember to mute again when you finish speaking)

Plan for today:

(1) advice for getting the most of your math major
(2) more practice problems

I have set up a discord server for this class.
The invitation is listed on the moodle or (directly):

https://discord.gg/PBNB4fQ

Advice to get the most of your math major at York

Short term advice: If you need help, don't be afraid to ask and seek it out. "This too shall pass"…the following advice is for the long term. Crisis aside, think about what will make you happy in life and work towards that goal.

1. **Go to your professor's office hours at least once a term**

2. **Learn/master a second language**

3. **Learn to program a computer**

4. **find opportunities to work on a research project**

5. **go on an exchange program or do an internship**

(f) Let $a_1, a_2, \ldots, a_n$ be positive real numbers. Show that

$$\left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} \frac{1}{a_i}\right) \geq n^2 \qquad \textbf{(*)}$$

Base case n=1: a_1*(1/a_1) = 1 >= 1^2
and so we know the statement is true for n=1

Assume that for some fixed n and positive numbers a_1,…, a_n that
(a_1 + a_2 + … + a_n)(1/a_1 + 1/a_2 + … + 1/a_n) >= n^2

Say that I have n+1 positive numbers a_1, a_2, ,…, a_n, a_{n+1}
(a_1 + a_2 + … + a_{n+1})(1/a_1 + 1/a_2 + … + 1/a_{n+1})
= ((a_1 + a_2 + … +a_n)+ a_{n+1})((1/a_1 + 1/a_2 + … +1/a_n) + 1/a_{n+1})
= (a_1 + a_2 + … +a_n)(1/a_1 + 1/a_2 + … +1/a_n) + a_{n+1}(1/a_1 + 1/a_2 + … +1/a_n)
+(a_1 + a_2 + … +a_n)1/a_{n+1} + a_{n+1}(1/a_{n+1})
>= n^2 + 1 + a_{n+1}/a_1 + a_{n+1}/a_2 + … + a_{n+1}/a_n
+ a_1/a_{n+1} + a_2/a_{n+1} + … + a_n/a_{n+1}
= n^2 + 1+ (a_{n+1}/a_1 + a_1/a_{n+1}) + (a_{n+1}/a_2+ a_2/a_{n+1}) + … + (a_{n+1}/a_n + a_n/a_{n+1})
>= n^2 + 1 + 2n (by the Lemma of problem #6 part (g))
= (n+1)^2

So I have shown: (a_1 + a_2 + … + a_{n+1})(1/a_1 + 1/a_2 + … + 1/a_{n+1}) >= (n+1)^2

Therefore, by the principle of mathematical induction, the equation (*)
is true for all integers n>=1

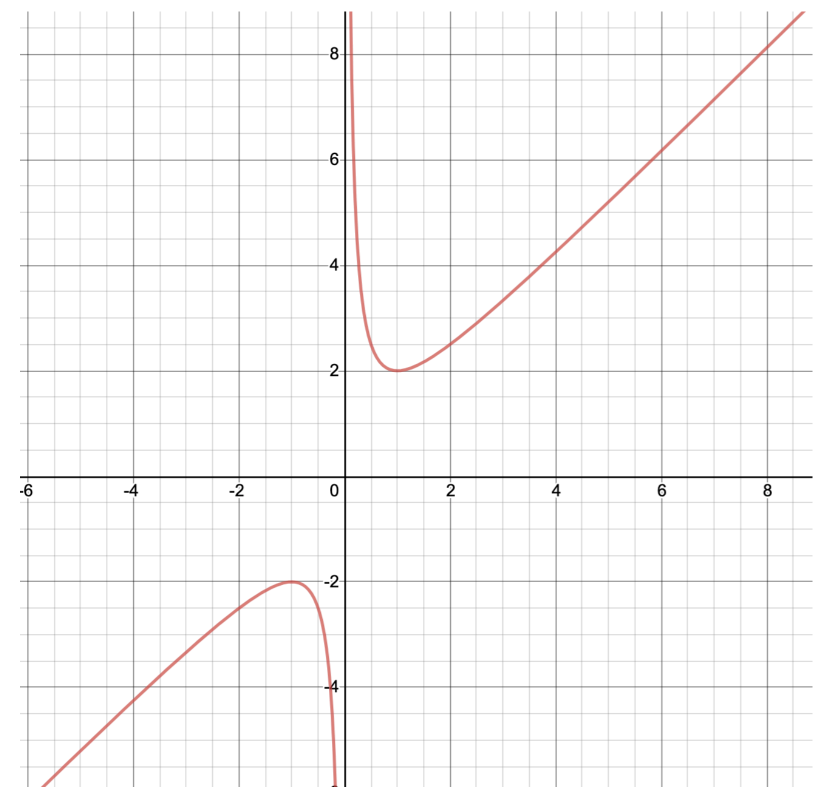
Lemma:

(g) If x is a positive real number, then $x + 1/x \geq 2$.

Proof: let f(x)=x+1/x be a function
and f’(x) = 1-1/x^2

so for 0 < x < 1
infinity > 1/x > 1
infinity > 1/x^2 > 1
-infinity < 1-1/x^2 < 0
f’(x) < 0 so f(x) is decreasing

for x>1 we have that x^2 > 1
and 1/x^2 < 1
1-1/x^2 > 0



The graph of f(x) = x + 1/x

at x=1, f’(x) = 0

by calculus we have that for positive values of x, f’(x) is decreasing 0<x<1,
f’(x) is increasing for x>1 and f’(x) = 0 (there is a local minimum) at x=1.

f(x) takes its minimum value for x>0 at the point x=1

f(x) >= 1+1/1= f(1) = 2 for all x>0

(g) Show that for $n \geq 1$, $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^n = \begin{bmatrix} \frac{5^n+3}{4} & 3 \cdot \frac{5^n-1}{4} \\ \frac{5^n-1}{4} & \frac{3 \cdot 5^n+1}{4} \end{bmatrix}$ **(*)**

Base case:

[2 3]^0 = [1 0] = [(5^0+3)/4 3*(5^0-1)/4]
[1 4] = [0 1] = [(5^0-1)/4 (3*5^0+1)/4]

Assume that equation (*) is true for some fixed n>=0

[2 3]^{n+1} = [2 3]^n * [2 3]
[1 4] = [1 4] * [1 4]

= [(5^n+3)/4 3*(5^n-1)/4] * [2 3]
[(5^n-1)/4 (3*5^n+1)/4] * [1 4]

= [2*(5^n+3)/4+1*3*(5^n-1)/4 3*(5^n+3)/4 + 4*3*(5^n-1)/4]
[2*(5^n-1)/4+1*(3*5^n+1)/4 3*(5^n-1)/4+4*(3*5^n+1)/4]

= [(5^{n+1}+3)/4 (15*5^n-3)/4]
[(5^{n+1}-1)/4 (15*5^n+1)/4]

= [(5^{n+1}+3)/4 (15*5^n-3)/4]
[(5^{n+1}-1)/4 (15*5^n+1)/4]

and this is equation (*) with n-> n+1.

And so by the principle of mathematical induction equation (*) holds for n>=0

(f) Find all values of $z \in \mathbb{C}$ such that $z^2\bar{z} = z$

first we know that if z=0 then z^2*bar_z = z = 0

assume that z is not 0, then if z^2*bar_z = z,
then z*bar_z = 1
if z = x + i*y, then
z*bar_z = (x+iy)(x-iy) = x^2 -i^2*y^2 = x^2 + y^2 = 1
therefore x^2 + y^2 = 1

The solution set to this equation are all the complex numbers whose points z =x+yi where (x,y) lie on the circle of radius 1 or z=0 + 0i

(h) For $n > 0$ and x and y are positive real numbers such that $xy > n^2$, then either $x > n$ or $y > n$

Restate this as contrapositive:
if x*y > n^2, then x>n or y>n
is logically equivalent to
if not(x>n or y>n), then not x*y>n^2
is logically equivalent to
if x<=n and y<=n , then x*y <= n^2
this statement is clearly true if x and y are both positive.

if x<n, and y>0,
x*y < n*y
if y<n and n>0,
y*n < n*n = n^2
by transitivity of <, then
x*y< n*y < n^2

1. For the following relations on the complex numbers (A) show that they are an equivalence relation and (B) find a description of the equivalence classes.

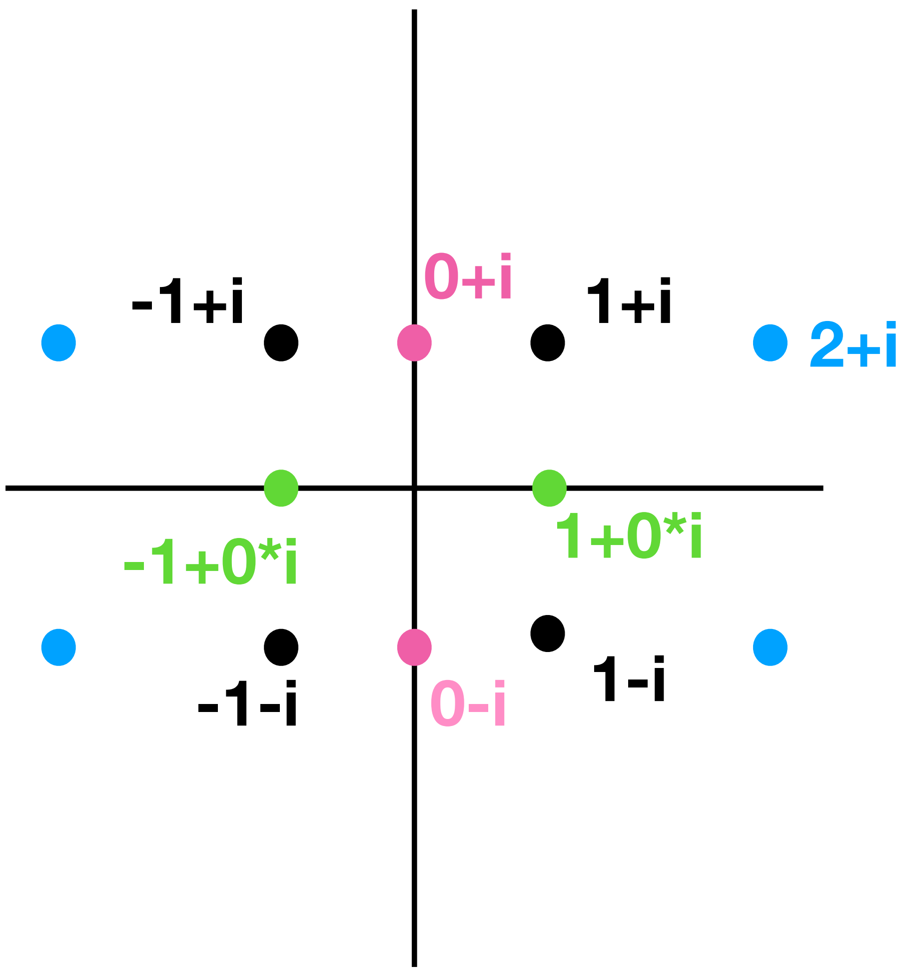
   (f) $R_6 = \{(z_1, z_2) \in \mathbb{C} \times \mathbb{C} : |Re(z_1)| = |Re(z_2)| \text{ and } |Im(z_1)| = |Im(z_2)|\}$

For any complex number z, |Re(z)|=|Re(z)| and |Im(z)| = |Im(z)|
so (z,z) is in R6 (therefore R6 is reflexive)

if (z1,z2) in R6, then |Re(z1)| = |Re(z2)| and |Im(z1)| = |Im(z2)|
so |Re(z2)|=|Re(z1) and |Im(z2)| = |Im(z1)|
therefore (z2,z1) in R6 and R6 is symmetric

if (z1,z2) and (z2,z3) in R6, then |Re(z1)|=|Re(z2)| and |Re(z2)|=|Re(z3)| so |Re(z1)| = |Re(z3)|. Similarly |Im(z1)|=|Im(z2)| = |Im(z3)| therefore (z1,z3) in R6 and therefore R6 is transitive.

Part (B) The equivalence classes are of the form
{a+bi, a-bi, -a+bi, -a-bi} for a,b > 0 positive real numbers
{a+0*i, -a+0*i} for a>0 positive real number
{0+b*i, 0-b*i} for b>0 a positive real number
and {0+0*i}
(a+b*i, a+bi), (a+b*i,a-b*i), (a+b*i, -a+b*i) and (a+b*i,-a-b*i) are all in R6
every element of the complex plane is in one of these equvialence classes



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what points are equivalent to 1+i in R6? That is what points (1+i,y) are in R6?
|Re(1+i)| = 1 = and we also have |Im(1+i)| = 1 = |Im(1-i)|
(1+i, 1-i) in R6
(1+i, -1-i), (1+i, -1+i) are also in R6 {1+i,1-i,-1+i, -1-i} is an equivalence class

(2+i, 2-i), (2+i, -2+i) and (2+i, -2-i) are in R6 {2+i,2-i,-2+i, -2-i}
is another equivalence class

{0+i, 0-i} is an equivalence class
{1,-1} {2,-2} are both equivalence classes