Hi All! I will have music playing in the background at least until 10am so that people can test their speakers.
Hey, if you hate my music you might want to turn off the volume until then. :)

If you need to play with the controls the menu next to “Start Video” has a option “Video Settings…” and then then there is an “Audio” tab where you can test speakers/microphone

When you come into the meeting your microphone should by default be mute

You may communicate by

1. the chat
2. signals of yes/no/faster/slower/thumbs up/thumbs down/etc.
3. you may unmute your microphone (but remember to mute again when you finish speaking)

4. I think that there is an option to annotate the screen (see if you can find the controls and draw on the screen)

Plan for today:

1. try two more examples of reflexive/symmetric/transitive
2. equivalence relations and classes on complex numbers

We will start with the following two relations on the set of integers (start thinking about them now!):

R9 = { (x,y) : x,y in Z such that x+1<2*y}
R10 = { (x,y) : x,y in Z such that x+y is odd}

is it reflexive? symmetric? transitive?

R9 = { (x,y) : x,y in Z such that x+1<2*y}

is it reflexive, symmetric and/or transitive? If so why or why not?

R9 is not reflexive because (1,1) is not in R9 because
1+1=2 and 2*1 = 2 and 1+1 is not less than 2

R9 is not symmetric, because (1,2) is in R9 because
1+1=2 and 2*2 = 4 and 2<4. 2+1=3 and 2*1=2 and 3 is not less than 2
(2,1) is not in R9

x=5, y=4, z=3 then x+1=6<2*y=8
and y+1=5 < 2*z = 6
x+1=6 is not less than 2*z=6
This is an example where (5,4) is in R9
(4,3) is in R9
and (5,3) is not in R9
so R9 is not transitive.

Recall:

A relation on a set D, is a set of pairs (x,y) where x and y are both in D.
A relation R is reflexive if (x,x) is in R for all x in D.
A relation R is symmetric iff for all x,y, in D, if (x,y) is in R, then (y,x) is in R.
A relation R is transitive iff for all x,y in D, if (x,y) and (y,z) is in R, then (x,z) is in R.

R10 = { (x,y) : x,y in Z such that x+y is odd}

is it reflexive, symmetric and/or transitive? If so why or why not?

x+x=2*x is even so (x,x) is not in R10 and R10 is not reflexive
1+1=2 and 2 is even so (1,1) is not in R10 and R10 is not reflexive

If (x,y) is in R10, then x+y is odd, y+x is also odd so (y,x) is in R10 and R10 is symmetric.

To test transitivity: (a,b)=(5,4), (b,c)=(4,3). Then (a,c)=(5,3), where a+c=8, which is not odd. So R10 is not transitive.

R10 = { (x,y) : if x,y in Z then x+y is odd}

Recall:

A relation on a set D, is a set of pairs (x,y) where x and y are both in D.
A relation R is reflexive if (x,x) is in R for all x in D.
A relation R is symmetric iff for all x,y, in D, if (x,y) is in R, then (y,x) is in R.
A relation R is transitive iff for all x,y in D, if (x,y) and (y,z) is in R, then (x,z) is in R.

A relation R is an equivalence relation if it is reflexive, symmetric and transitive

If *a* is an element of D, then the equivalence class of *a* is

[a] := { b : b in D such that (a,b) in R }

Fact 1: every element of D is in some equivalence class

Proof: (x,x) is in R for every x in D because R is reflexive, so x in [x]

Fact 2: every two equivalence classes are disjoint or equal
if two equivalence classes have (at least) one element in common then they are equal
if there is an element z in D such that z in [x] and z in [y], then [x]=[y]

Proof: Assume that there is an element z in D, such that z in [x] and z in [y]
since z in [x], then (x,z) in R and z in [y] so (y,z) in R
I will show that [x] is a subset of [y] and [y] is a subset [x]
take a w in [x] and show that w is also in [y]
(x,w) in R
Since (x,z) in R then (z,x) is in R because R is symmetric
since (y,z) in R and (z,x) is in R, then (y,x) is in R because R is transitive
since (y,x) in R and (x, w) is in R, then (y,w) is in R therefore w in [y]
and I can conclude that since w is arbitrary, then [x] is a subset of [y]
The same argument show that [y] is a subset of [x] because [x] and [y]
are interchangable.
therefore [x]=[y]

The following are examples of equivalence relations on the complex numbers

$$R_1 = \{(x, x) : x \in \mathbb{C}\}$$

$$R_2 = \{(x, y) : Re(x) = Re(y), x, y \in \mathbb{C}\}$$

$$R_3 = \{(x, y) : Im(x) = Im(y), x, y \in \mathbb{C}\}$$

$$R_4 = \{(x, y) : |x| = |y|, x, y \in \mathbb{C}\}$$

$$R_5 = \{(x, y) : ax = y \text{ for some } a > 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

$$R_6 = \{(x, y) : ax = y \text{ for some } a \neq 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

if x = a+bi where a and b are real
then Re(x) = a and Im(x) = b

Step 1: show that the following relation on complex numbers is an equivalence relation

$$R_2 = \{(x, y) : Re(x) = Re(y), x, y \in \mathbb{C}\}$$

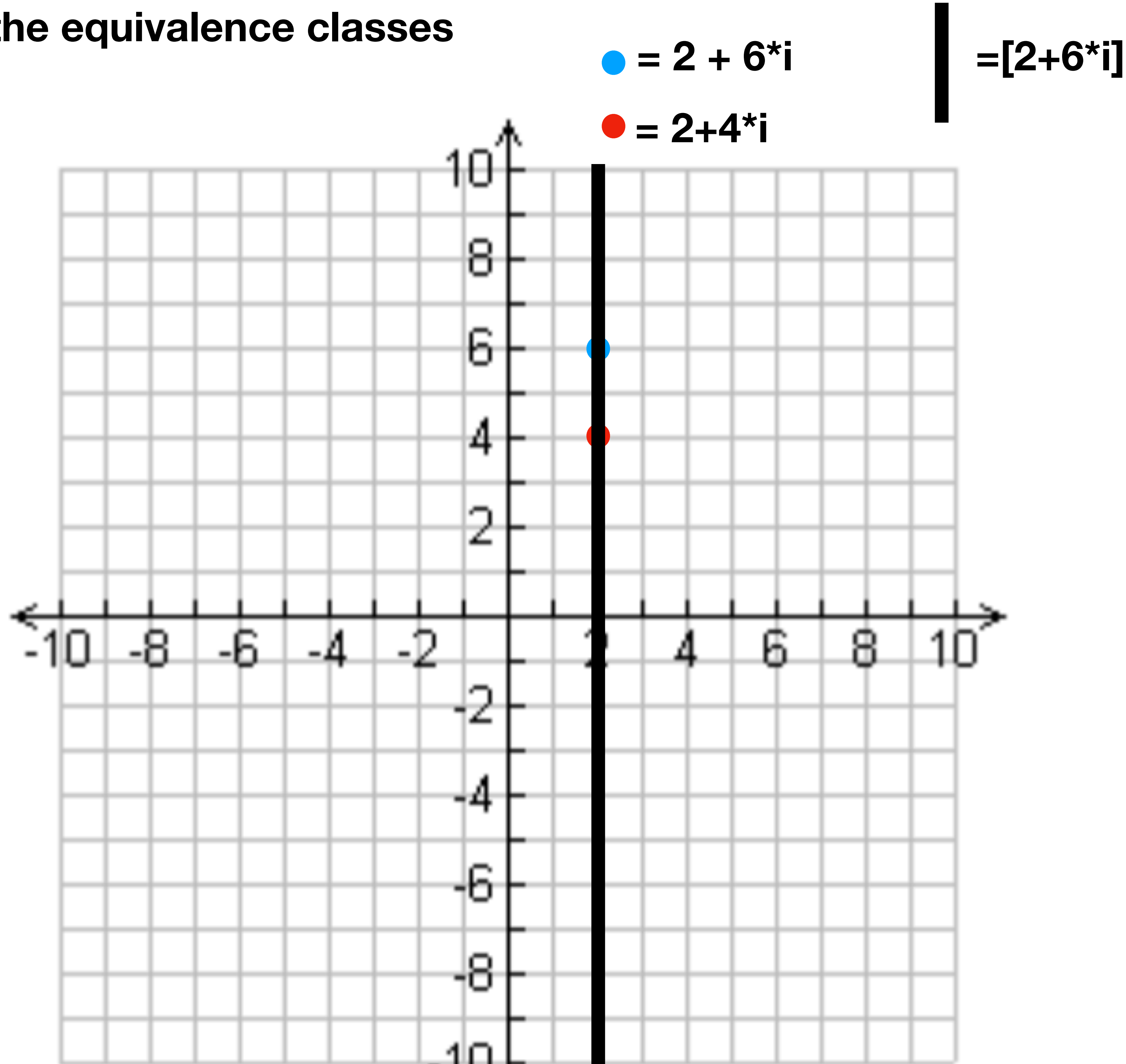
R2 is reflexive because Re(x) = Re(x) so (x,x) is in R2

R2 is symmetric because if (x,y) in R2 then Re(x)=Re(y) so Re(y)=Re(x) therefore (y,x) is in R2

R2 is transitive because if (x,y) and (y,z) are in R2 then Re(x)=Re(y) and Re(y)=Re(z) But this means that Re(x)=Re(z) so (x,z) is in R2.

Step 2: identify the equivalence classes (next page)

Step 2: identify the equivalence classes



The equivalence classes of R2 are the vertical lines in the complex plane

$$R_2 = \{(x, y) : Re(x) = Re(y), x, y \in \mathbb{C}\}$$

For example: (2+6*i,2+4*i) in R2