

**Hi All! I will have music playing in the background at least until 10am so that people can test their speakers.**

**Hey, if you hate my music you might want to turn off the volume until then. :)**

If you need to play with the controls the menu next to “Start Video” has a option “Video Settings...” and then then there is an “Audio” tab where you can test speakers/microphone

When you come into the meeting your microphone should by default be mute

You may communicate by

1. the chat
2. signals of yes/no/faster/slower/thumbs up/thumbs down/etc.
3. you may unmute your microphone (but remember to mute again when you finish speaking)
4. I think that there is an option to annotate the screen (see if you can find the controls and draw on the screen)

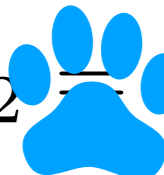
Plan for today:

- (1) finish with equivalence relations (one more example from complex numbers)
- (2) summation notation

**We will start with an an example from the list of equivalence relations on complex numbers: (a) prove it is an equivalence relation  
(b) identify the equivalence classes**


The following are examples of equivalence relations on the complex numbers

$$R_1 = \{(x, x) : x \in \mathbb{C}\}$$

$$R_2 \text{  } \{(x, y) : \operatorname{Re}(x) = \operatorname{Re}(y), x, y \in \mathbb{C}\}$$

$$R_3 = \{(x, y) : \operatorname{Im}(x) = \operatorname{Im}(y), x, y \in \mathbb{C}\}$$

$$R_4 = \{(x, y) : |x| = |y|, x, y \in \mathbb{C}\}$$

$$R_5 \text{  } \{(x, y) : ax = y \text{ for some } a > 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

$$R_6 \text{  } \{(x, y) : ax = y \text{ for some } a \neq 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

**if  $x = a+bi$  where  $a$  and  $b$  are real  
then  $\operatorname{Re}(x) = a$  and  $\operatorname{Im}(x) = b$**

**Step 1: show that the following relation on complex numbers is an equivalence relation**

$$R_6 = \{(x, y) : ax = y \text{ for some } a \neq 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

**R6 is reflexive because if  $x$  is not 0, then  $1 \cdot x = x$  where  $a=1$  not equal to 0 so  $(x,x)$  in R6  
 $0 \cdot a = 0$  for any  $a$  not equal to 0 so  $(0,0)$  is in R6 also**

**if  $(x,y)$  in R6, then  $a \cdot x = y$  for some  $a$  not equal to 0**

**if we let  $b=1/a$ , then**

$$x = y/a = (1/a) \cdot y = b \cdot y$$

**$b \cdot y = x$  for some  $b$  not equal to 0**

**so  $(y,x)$  in R6 and R6 is symmetric**

**if  $(x,y)$  and  $(y, z)$  are in R6, then there is  $a_1$  and  $a_2$  both not equal to 0 such that  $a_1 \cdot x = y$  and  $a_2 \cdot y = z$**

$$a_2 \cdot y = a_2 \cdot (a_1 \cdot x) = z$$

**if I let  $a_3 = a_1 \cdot a_2$  then  $a_3$  is not equal to 0**

$$\text{and } a_3 \cdot x = a_2 \cdot a_1 \cdot x = z$$

**that implies that  $(x,z)$  in R6**

**therefore R6 is transitive**

**Therefore R6 is an equivalence relation**

**Step 2: identify the equivalence classes (next page)**

Step 2: identify the equivalence classes

● =  $3 + 6i$

● =  $-3 - 6i$

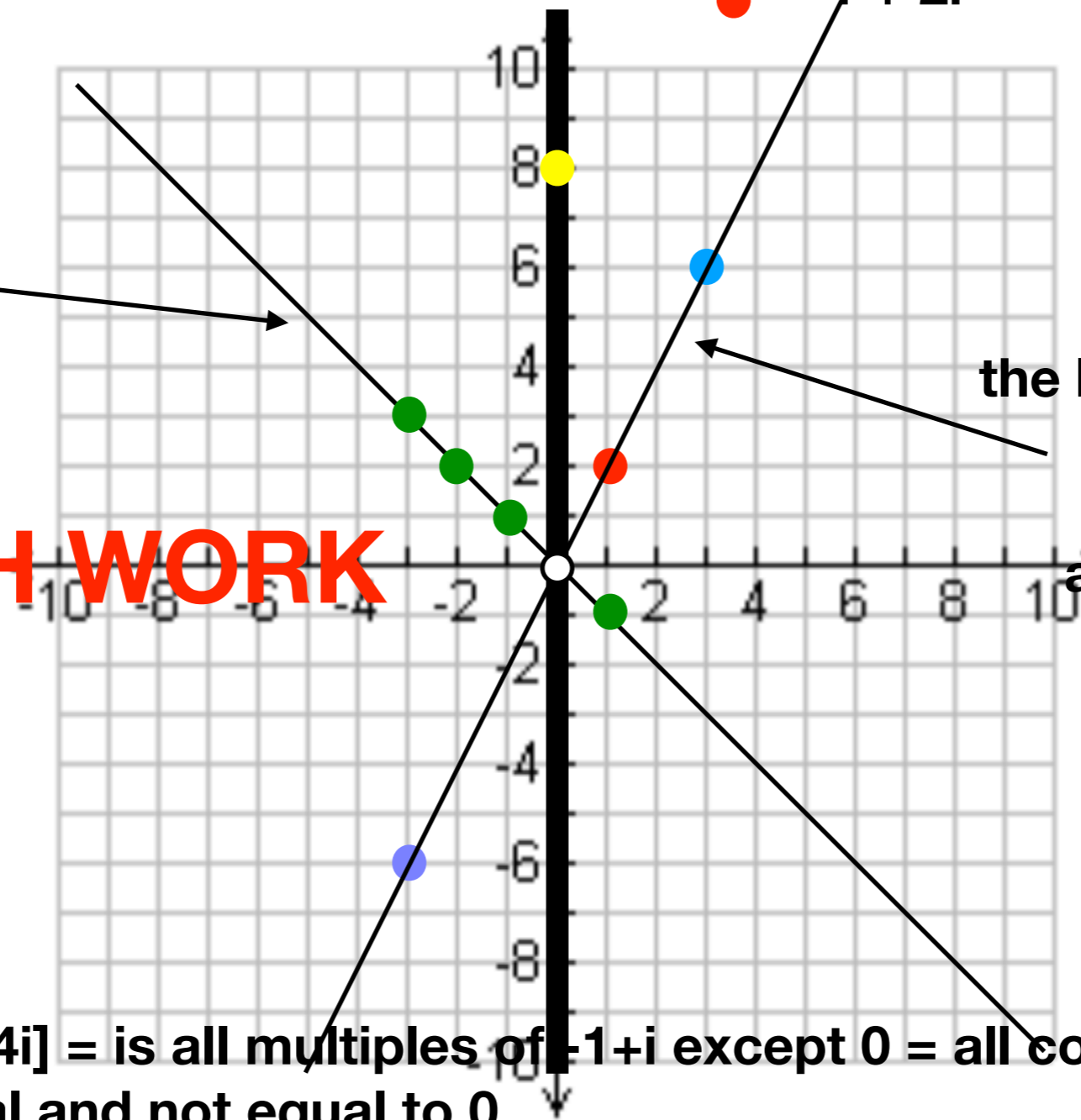
● =  $1 + 2i$   
● =  $6 + 12i$

● =  $-3 + 3i$

slope =  $-1 = \text{rise/run}$   
y-intercept = 0

the line which passes through  $1+2i$  is a line with slope = rise/run =  $2/1$  and y-intercept = origin

**SCRATCH WORK**



$[-3+3i] = [-1+i] = [-4+4i]$  = is all multiples of  $-1+i$  except 0 = all complex numbers of the form  $x-xi$  with "x" real and not equal to 0

$[1+2i]$  = is all multiples of  $1+2i$  except 0

$1+2i$  is in  $[3+6i]$

$(1+2i, 3+6i)$  in  $R_6$  because  $3*(1+2i) = 3+6i$  shows that  $3+6i$  in  $[1+2i]$

$(3+6i, 1+2i)$  in  $R_6$  because  $R_6$  is symmetric (or because  $1/3*(3+6i) = 1+2i$ )

$$R_6 = \{(x, y) : ax = y \text{ for some } a \neq 0, a \in \mathbb{R}, x, y \in \mathbb{C}\}$$

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**Fact 1: The equivalence classes are all the lines which pass through the origin of the complex plane and don't include the origin itself.**

**Proof: let  $z = x+yi$  where  $x$  and  $y$  are real be a complex number. The equivalence class of  $[z]$  are all the points of the form  $a \cdot z$  for some  $a$  not equal to 0. These are the complex numbers  $ax + ay i = az$  these are the points which are on a line with slope =  $ay/ax = y/x$  and  $y$ -intercept equal to the origin (that is these are the points which lie on a slope with constant slope  $y/x$ ).**

**if  $x=0$ , then  $z= yi$  and the equivalence class  $[z] = [yi] =$  this is the vertical line passing through the origin with a hole at the origin.**

**Fact 2:**

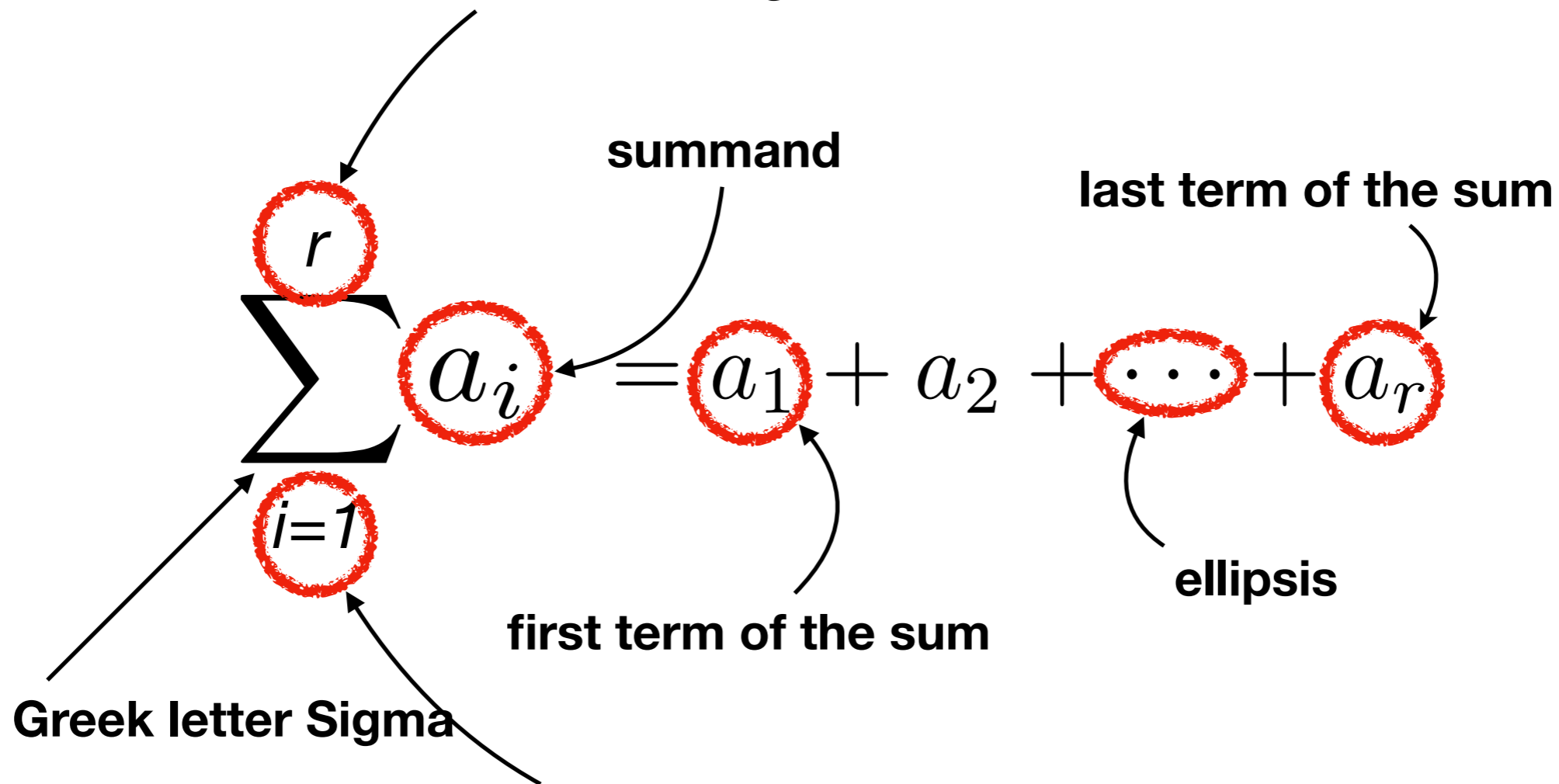
**There is one more class  $[0]$  which consists only of the origin.**

# Anatomy of a summation

upper limit of the summation

$r$  is an integer (usually  $r \geq$  the first value of the index of the sum)

there should be something here if the sum is finite



lower limit of the summation

$i$  is the index summation

an infinite sum is often called a "series"