Hi All! I will have music playing in the background at least until 10am so that people can test their speakers.
Hey, if you hate my music you might want to turn off the volume until then. :)

If you need to play with the controls the menu next to “Start Video” has a option “Video Settings…” and then then there is an “Audio” tab where you can test speakers/microphone

When you come into the meeting your microphone should by default be mute

You may communicate by

1. the chat
2. signals of yes/no/faster/slower/thumbs up/thumbs down/etc.
3. you may unmute your microphone (but remember to mute again when you finish speaking)

Plan for today:

(1) more summation notation
(2) discussion about practice for final

A practice for the final has been posted on the web page and moodle
Also: do you know how to do every question on the midterm? practice midterm? practice midterms and finals from previous semesters (all my web pages are public!)?
We will spend a while talking about summation notation, but then we can start review.

Anatomy of a summation

upper limit of the summation
r is an integer (usually r>= the first value of the index of the sum)
there should be something here if the sum is finite

summand

last term of the sum

$$\sum_{i=1}^{r} a_i = a_1 + a_2 + \cdots + a_r$$

first term of the sum

ellipsis

Greek letter Sigma

lower limit of the summation
i is the index summation

Advantage:
notation is much more compact

Disadvantage:
hard to read

the expression on the
left hand side of the equation is
not very unique

an infinite sum is often called a “series”

This is an infinite sum. The x is a variable
an infinite sum is sometimes called a "series".

$$\sum_{n\geq 0} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots \qquad = \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{k} a_n = \sum_{n=5}^{k+5} a_{n-5} \quad = a_0 + a_1 + a_2 + \cdots + a_k$$

in both cases the variable n takes on k+1 values
in other words: there are k+1 terms in the sum

$$\sum_{n=1}^{n} a_n$$

This is bad notation. There is a typo here because n cannot appear as the index of the sum and at the same time appear in the upper limit.
We cannot tell what the upper limit is if n is changing

index variable = n. the index variable can occur in the blue area only

$$\sum_{n=0}^{k} \frac{k}{n} \log\Big(1 - \frac{x}{n}\Big)$$

Very important that all of the summands are defined for each of the values that the index variable n takes on.
Note here: for n=0 the summand isn't defined

$$\cos(0) + \cos(2\pi/n) + \cos(4\pi/n) + \cdots + \cos(2(n-1)\pi/n) = 0$$

this sum has n terms in total

second term not correct

A. $$\sum_{r=0}^{n-1} \cos(2r\pi/(r+1)) = 0$$

makes no sense without the upper limit

B. $$\sum_{r=0} \cos(2r\pi/n) = 0$$

C. $$\sum_{r=0}^{n-1} \cos(2r\pi/n) = 0$$

first term not correct

D. $$\sum_{r=0}^{n} \cos(2(r-1)\pi/n) = 0$$

last term not correct

E. $$\sum_{r=0}^{n} \cos(2r\pi/n) = 0$$

F. $$\sum_{r=1}^{n} \cos(2(r-1)\pi/n) = 0$$

G. None of the above

H. All of the above

I. I don't care, stop with the summation notation already

=parts of the expression that change in the summand

=parts of the expression that stay the same in the summand

$$\sum_{i=1}^{k} i$$

$$\sum_{i=1}^{k} i = 1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

\sum_{i=1}^k i

$$\sum_{i=1}^{k} i^3 = 1^3 + 2^3 + \cdots + k^3 = (1 + 2 + \cdots + k)^2$$

\sum_{i=1}^k i^3

$$\sum_{i=1}^{k+1} 2^{2i-1} = \sum_{i=0}^{k} 2^{2i+1} = 2^1 + 2^3 + 2^5 + \cdots + 2^{2k+1}$$

\sum_{i=0}^{k} 2^{2i+1}

\sum_{i=1}^{k+1} 2^{2i-1}

$\sum_{i=0}^{2k+1} 2^{2k+1}$ = 2^{2k+1} + 2^{2k+1} + 2^{2k+1} +…+ 2^{2k+1}

$\sum_{i=1}^{2k+1} 2^{i}$ = 2 + 2^2 + 2^3 + 2^4 +… 2^{2k+1}

$\sum_{i=1}^{2k+1} i$ = 1 + 2+ 3+ … + 2k+1

$\sum_{i=0}^{2k+1} 2^{2i+1}$ = 2^1 + 2^3 + 2^5 + … + 2^{4k+3}

$$\sum_{i=1}^{k} i(i+1)(i+2) = 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + \cdots + k(k+1)(k+2)$$

summation needs to encode exactly k terms

\sum_{i=1}^{k} i(i+1)(i+2)

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\sum_{i=1}^{\infty} ix^{i -1}
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\sum_{i=0}^{\infty} (i+1)x^i
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$$\sum_{i=1}^{\infty} ix^{i-1} = \sum_{i=0}^{\infty} (i+1)x^i = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

$$\sum_{i=0}^{\infty} kx^{i-1}$$ = kx^{-1} + k x^0 + k x^1 + k x^2 + …

\sum_{i=0}^{\infty} kx^{i-1}

$$\left(\sum_{i=1}^{\infty} (i)x^i\right) - 1$$ = (1x^1 + 2x^2 + 3x^3 + …)-1

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\left(\sum_{i=1}^{\infty} (i)x^i \right) -1
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$$\sum_{i=1}^{\infty} \left((i)x^i - 1\right)$$ = (1x-1) + (2x^2-2) + (3x^3-3) + ….

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\sum_{i=1}^{\infty} \left((i)x^i -1\right)
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correction (added Apr 13): this should read: (1x-1) + (2x^2-1) + (3x^3-1) + …

$$\frac{k}{1}\log(1-x)+\frac{k}{2}\log(1-x/2)+\frac{k}{3}\log(1-x/3)+\cdots+\log(1-x/k)$$

$$\sum_{n=1}^{k}\frac{k}{n}log(1-\frac{x}{n})$$ **= k/1*log(1-x/1) + k/2*log(1-x/2) + k/3*log(1-x/3) + … +k/k*log(1-x/k)**

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\sum_{n=1}^{k} \frac{k}{n}log(1-\frac{x}{n})
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capital greek letter Sigma $$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$ $$\sum_{i=1}^{0} a_i = 0$$

capital greek letter Pi $$\prod_{i=1}^{n} a_i = a_1 a_2 \cdots a_n$$ $$\prod_{i=1}^{0} a_i = 1$$

$$\forall i \in \{1, 2, \ldots, n\}, a_i = a_1 \text{ and } a_2 \text{ and } \cdots \text{ and } a_n$$

$$(\forall i \in \{\}, a_i) = true$$

$$\exists i \in \{1, 2, \ldots, n\}, a_i = a_1 \text{ or } a_2 \text{ or } \cdots \text{ or } a_n$$

$$(\exists i \in \{\}, a_i) = false$$

WHAT HAPPENS WHEN n=0???

$$\left(\sum_{i=1}^{n-1} a_i\right) + a_n = \left(\sum_{i=1}^{n-2} a_i\right) + a_{n-1} + a_n = \left(\sum_{i=1}^{n} a_i\right)$$

$$\left(\sum_{i=1}^{1} a_i\right) + a_2 + \cdots + a_{n-1} + a_n =$$

$$\left(\sum_{i=1}^{0} a_i\right) \cancel{+ a_1 + a_2 + \cdots + a_{n-1} + a_n} = \cancel{a_1 + a_2 + \cdots + a_n}$$

$$\sum_{i=1}^{0} a_i = 0$$

$$\left(\prod_{i=1}^{0} a_i\right) \cancel{a_1}\cancel{a_2}\cdots\cancel{a_{n-1}}\cancel{a_n} = \cancel{a_1}\cancel{a_2}\cdots\cancel{a_n} \quad \mathbf{1}$$

$$\prod_{i=1}^{0} a_i = 1$$

$$\forall i \in \{1, 2, \ldots, n\}, a_i = a_1 \text{ and } a_2 \text{ and } \cdots \text{ and } a_n$$

$$(\forall i \in \{\}, a_i) \text{ and } a_1 \text{ and } a_2 \text{ and } \cdots \text{ and } a_n = a_1 \text{ and } a_2 \text{ and } \cdots \text{ and } a_n$$

$$(\forall i \in \{\}, a_i) = true$$

for all i in the empty set, a_i is true (because there are no i in the empty set)

$$\exists i \in \{1, 2, \ldots, n\}, a_i = a_1 \text{ or } a_2 \text{ or } \cdots \text{ or } a_n$$

$$(\exists i \in \{\}, a_i) \text{ or } a_1 \text{ or } a_2 \text{ or } \cdots \text{ or } a_n = a_1 \text{ or } a_2 \text{ or } \cdots \text{ or } a_n$$

$$(\exists i \in \{\}, a_i) = false$$

there does not exist an i in the empty set that makes a_i true (so we say “exists” is false)