

Hi All! I will have music playing in the background at least until 10am so that people can test their speakers.

Hey, if you hate my music you might want to turn off the volume until then. :)

If you need to play with the controls the menu next to “Start Video” has a option “Video Settings...” and then then there is an “Audio” tab where you can test speakers/microphone

When you come into the meeting your microphone should by default be mute

You may communicate by

1. the chat
2. signals of yes/no/faster/slower/thumbs up/thumbs down/etc.
3. you may unmute your microphone (but remember to mute again when you finish speaking)

Plan for today:

(1) practice for final

**I have set up a discord server for this class.
The invitation is listed on the moodle or (directly):**

<https://discord.gg/PBNB4fQ>

3. Let n be a positive integer and a, b, c, d be integers. Provide a proof of the following statements.

(d) if $a \equiv b \pmod{n}$ then $a + c \equiv b + c \pmod{n}$

Assume $a \equiv b \pmod{n}$

then n divides $(a-b)$

so n divides $((a+c) - (b+c)) = a-b$

therefore $a+c \equiv b+c \pmod{n}$

(i) if $a \cdot b \equiv a \cdot c \pmod{n}$ and $\gcd(a, n) = 1$, then $b \equiv c \pmod{n}$

Assume $a \cdot b \equiv a \cdot c \pmod{n}$ and $\gcd(a, n) = 1$

so n divides $(ab - ac) = a(b-c)$

and there exists k and ℓ integers such that $k \cdot a + \ell \cdot n = 1$

$k \cdot a = 1 - \ell \cdot n$

since n divides $a \cdot (b-c)$ then it also divides $k \cdot a \cdot (b-c) = (1 - \ell \cdot n) \cdot (b-c) = b-c - \ell \cdot n \cdot (b-c)$

therefore $n \cdot t = b-c - \ell \cdot n \cdot (b-c)$ for some integer t

and we conclude that $n \cdot t + \ell \cdot n \cdot (b-c) = b-c = n \cdot (t + \ell \cdot (b-c))$

and so n divides $b-c$

therefore $b \equiv c \pmod{n}$

(g) Find all values of $z \in \mathbb{C}$ such that $|z + 3i| = 3|z|$

$$z = x + iy$$

$$|z+3i| = \sqrt{x^2 + (y+3)^2} = 3 \sqrt{x^2 + y^2} = 3|z|$$

$$x^2 + (y+3)^2 = 9(x^2 + y^2)$$

$$x^2 + y^2 + 6y + 9 = 9x^2 + 9y^2$$

$$0 = 8x^2 + 8y^2 - 6y - 9$$

$$9 + 8a^2 = 8x^2 + 8(y^2 - 6/8y + a^2)$$

$$(y - a)^2 = y^2 - 2ay + a^2 \implies 2a = 6/8 \implies a = 6/16 = 3/8$$

$$9 + 8(3/8)^2 = 8x^2 + 8(y - 3/8)^2$$

$$(9 + 8(3/8)^2)/8 = x^2 + (y - 3/8)^2$$

this is a circle with center $(0, 3/8)$ and radius $\sqrt{(9 + 8(3/8)^2)/8}$

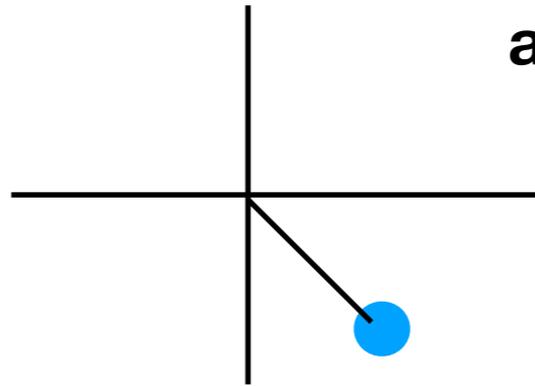
completing the square::: $y^2 + b*y$ then we guess that

$$y^2 + b*y = y^2 + 2*a*y + a^2 - a^2 = (y+a)^2 - a^2$$

(a) Find all values of $n > 0$ such that $(1 - i)^n$ is imaginary.

m 2+4*m

1 6
 2 10
 3 14
 4 18
 5 22
 6 26
 7 30
 8 34
 9 38
 10 42
 11 46
 12 50
 13 54
 14 58
 15 62
 16 66
 17 70



$|1-i| = \sqrt{1^2 + 1^2} = \sqrt{2}$
 angle = $3\pi/2 + \pi/4 = 7\pi/4 = -\pi/4$

$1-i = \sqrt{2} * e^{i*7\pi/4}$

$(1-i)^n = \sqrt{2}^n * e^{n*i*7\pi/4}$

to say this is a completely imaginary number then the angle will need to be $\pi/2 + m*2\pi$ or $3\pi/2 + m*2\pi$
 $\pi/2 + m\pi$ where m is some integer

so we are asking when: $n*7\pi/4 = \pi/2 + m\pi$
 $n*7 = 2 + 4*m$ looking for m and n that solve this equation
 $7*n - 2 = 4*m$ so 4 divides $7*n - 2$

$7*n \equiv 2 \pmod{4}$

$7 \equiv -1 \pmod{4}$

so $-n \equiv 2 \pmod{4}$ $7 - (-1) = 8 = 2*4$

$n \equiv -2 \pmod{4}$

n is of the form $4*ell - 2$ for some integer ell

n	7*n
1	7
2	14
3	21
4	28
5	35
6	42
7	49
8	56
9	63

$D = \{1,2,3,4\}$

anti-reflexive

anti-symmetric

circular

$R = \{\}$

(e) if $a \equiv b \pmod{n}$ then $a \cdot c \equiv b \cdot c \pmod{n}$

so $-n \equiv 2 \pmod{4}$

$n \equiv -2 \pmod{4}$

$7 \equiv -1 \pmod{4}$

because 4 divides $(7 - (-1)) = 8 = 2 \cdot 4$