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The Mathematics of Voting

by Florin Diacu

Did your vote in the recent federal election convey your will? Think of the ballot. You had to say: I like candidate *x* and I reject all the others. You may have wished to make *y* your second choice or tell that *z* was unacceptable. Unfortunately you couldn't. Our system is not that flexible. Let us improve it then. But how?

In a recent lecture given at the University of Victoria, Donald Saari, a distinguished mathematician from the University of California at Irvine, addressed this issue and showed the advantages and drawbacks of different democracies. Is there a best voting system, and if so, how good is it? The problem is difficult. An entire branch of mathematics is researching it today. Let us follow some ideas and see how we can use them.

The beginnings of voting are lost in history. Written sources attest the existence of voting procedures in Antiquity and all through the Middle Ages. Confusion in choosing the right system was common. In 1130, for example, the ambiguity of voting led to the election of two Popes, an event that created a rift within the Catholic Church.

In 1770 the French mathematician Jean-Charles Borda (1733-1799) proposed a new rule. He asked that voters rank the candidates and that points are accordingly assigned. For example, in a 3 candidate election, the first ranked on a ballot received 3 points, the second obtained 2, and the third got 1. The candidate with more points won.

But we could use different point rules: 6 for the first place, 5 for the second, and 0 for the third; or 10 for the first, 2 for the second, and 1 for the third. In fact our present system gives 1 for the first place and 0 for the others. Which one is better?

Though we can see some pros and cons in each case, it is hard to choose the best. But mathematicians found the answer. They have bad and good news for us. The bad news: Borda's count 3,2,1 is not ideal; it can still lead to distorted results. The good news: within the point method, the Borda count is by far the best. Moreover, our voting rule 1,0,0 is the worst; it gives the least amount of information about what voters want and can yield results that speak against the people's will.

This becomes clear from the following examples. In 1970 the centre-right candidate Buckley won the New York senate election even though more than 60% of the votes went for either of the two centre-left candidates. A less obvious but even more disturbing case is the recent Bush-Gore race. If those voting for Nader could have made Gore their second choice (which is a reasonable assumption), the democrats would have won without trouble, as the popular vote suggests.

But there are more complicated systems. The run-off method, for example, uses the 1,0,0 point rule in

combination with several rounds of vote. Only a more than 50% support makes the winner. Otherwise the last candidate is dropped and the vote is repeated. An alternative is to exclude all but the first two candidates and vote a second time.

This system, however, has its flaws too. The first version takes too long to be efficient in a national election, whereas the second can bring weird outcomes. In the 26 November 2000 election for the Romanian Presidency, this method led to a run-off between a left wing extremist and a right wing one. The centre vote had been split among several candidates.

The only simple and efficient method that in most cases expresses the will of the majority is the Borda count. Ranking the candidates and assigning a balanced rule of points, as in the 3,2,1 example, would make our elections fairer. We only need to implement this rule. Its time has arrived.

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