THE JARGON OF PROBABILITY

EXPERIMENT

ELEMENTARY OUTCOME, SAMPLE POINT

SAMPLE SPACE

EVENT

ASSOCIATED FIELD OF EVENTS

PROBABILITY

CONDITIONAL PROBABILITY

RANDOM VARIABLE

EXPECTATION

CONDITIONAL EXPECTATION

DEPENDENCE

INDEPENDENCE

THE JARGON OF PROBABILITY

EXPERIMENT, RANDOM VARIABLES

This refers to an activity, not necessarily scientific, which involves the the production of data some of which are “random”. We denote an experiment by $\mathcal{E}$ and the data by $X, Y, Z, \ldots$. The latter are usually referred to as the *RANDOM VARIABLES* associated with $\mathcal{E}$.

RANDOM, SAMPLE SPACE, PROBABILITIES

We use the word *RANDOM* whenever the data $X, Y, Z, \ldots$ we are studying are produced by such an intricate mechanism that all we know about them is

(1) The range of possible values that $X, Y, Z, \ldots$ may take. This range is usually referred to as the *SAMPLE SPACE* and denoted by the symbol Ω .

(2) Certain positive numbers called *PROBABILITIES* which numerically express our “confidence ” that $X, Y, Z, \ldots$ fall in chosen subsets of the sample space Ω .

ELEMENTARY OUTCOME, SAMPLE POINT

An individual outcome of the experiment $\mathcal{E}$ is usally referred to as an *ELEMENTARY OUTCOME* or *SAMPLE POINT*. Mathematically this is just an element of the sample space Ω .

EVENT

Mathematically an *EVENT* is just a subset of Ω . We say that $\mathcal{E}$ “resulted in the event A” or that “ A has occurred” if the outcome falls in the subset A.

FIELD OF EVENTS

The collection of events associated with our experiment $\mathcal{E}$ is usually denoted by $\mathcal{F}$. In other words, $\mathcal{F}$ denotes the collection of subsets of the sample space Ω that are of special interest in our study. For mathematical reasons $\mathcal{F}$ is assumed to be closed under the set operations of *intersection*, *union* and *complementation*. The two subsets ϕ and Ω are always included in $\mathcal{F}$.

PROBABILITY MEASURE

Our experiment $\mathcal{E}$ associates to each event A of F a number $P[A]$ in the interval $[0,1]$ which is reflects our confidence that the outcome falls in A. We refer to $P[A]$ as the " probability of A". Note that we should have $P[\Omega] = 1$ and that if A and B are mutually esclusive events then

$$P[A \;+\; B] \;=\; P[A] \;+\; P[B]$$

A set function with these properties is usually referred to as a *PROBABILITY MEASURE*.

EXPECTATION OF A RANDOM VARIABLE

Any function of the outcome of our experiment can be referred to as a *RANDOM VARIABLE*. Mathematically, a random variable is simply a function on the sample space. If the events A_1 , A_2 , ... , A_k are mutually esclusive and decompose Ω , and the random variable X takes the value x_i when A_i occurs then the expression

$$E[X] \;=\; x_1 P[A_1] \;+\; x_2 P[A_2] \;+\; \cdots \;+\; x_k P[A_k]$$

is referred to as the *EXPECTATION OF X*. If we repeat $\mathcal{E}$ a very large number of times, and average out the successive values of X we get, then we should **expect** the resulting average to be close to $E[X]$.

CONDITIONAL PROBABILITY

If **A** and **B** are events the ratio

$$P[A|B] \quad = \quad \frac{P[A \cap B]}{P[B]}$$

is usually referred to as the *CONDITIONAL PROBABILITY OF* **A** *GIVEN* **B** The concept arises as follows. Given the event B we can construct a new experiment $\mathcal{E}_B$ by carrying out $\mathcal{E}$ and recording its outcome **only** when it falls in **B**. We can argue that the probability of **A** under $\mathcal{E}_B$ will is $\frac{P[A \cap B]}{P[B]}$ where $P[A \cap B]$ and $P[B]$ are the probabilities of $\mathbf{A} \cap \mathbf{B}$ and **B** under $\mathcal{E}$. We shall refer to $\mathcal{E}_B$ as $\mathcal{E}$ *CRIPPLED by* **B**.

CONDITIONAL EXPECTATION OF A RANDOM VARIABLE

Given an event B, if we carry out the crippled experiment $\mathcal{E}_B$ instead of $\mathcal{E}$, then all the probabilities change and so do all expectations. If X is a random variable and the events A_1 , A_2 , ... , A_k decompose Ω as before then expression

$$E[X|B] \quad = \quad x_1 P[A_1|B] \quad + \quad x_2 P[A_2|B] \quad + \quad \cdots \quad + \quad x_k P[A_k|B]$$

gives the expected value of X under $\mathcal{E}_B$. We refer to it as the *CONDITIONAL EXPECTATION OF X GIVEN B*.

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DEPENDENCE

The random variable Y is said to be *DEPENDENT* upon the random variable X if and only if Y is a function of X. Similarly we say that Y is dependent upon $X_1, X_2, \ldots, X_n$ if for some function $f(x_1, x_2, \ldots, x_n)$ we have

$$Y = f(X_1, X_2, \ldots, X_n)$$

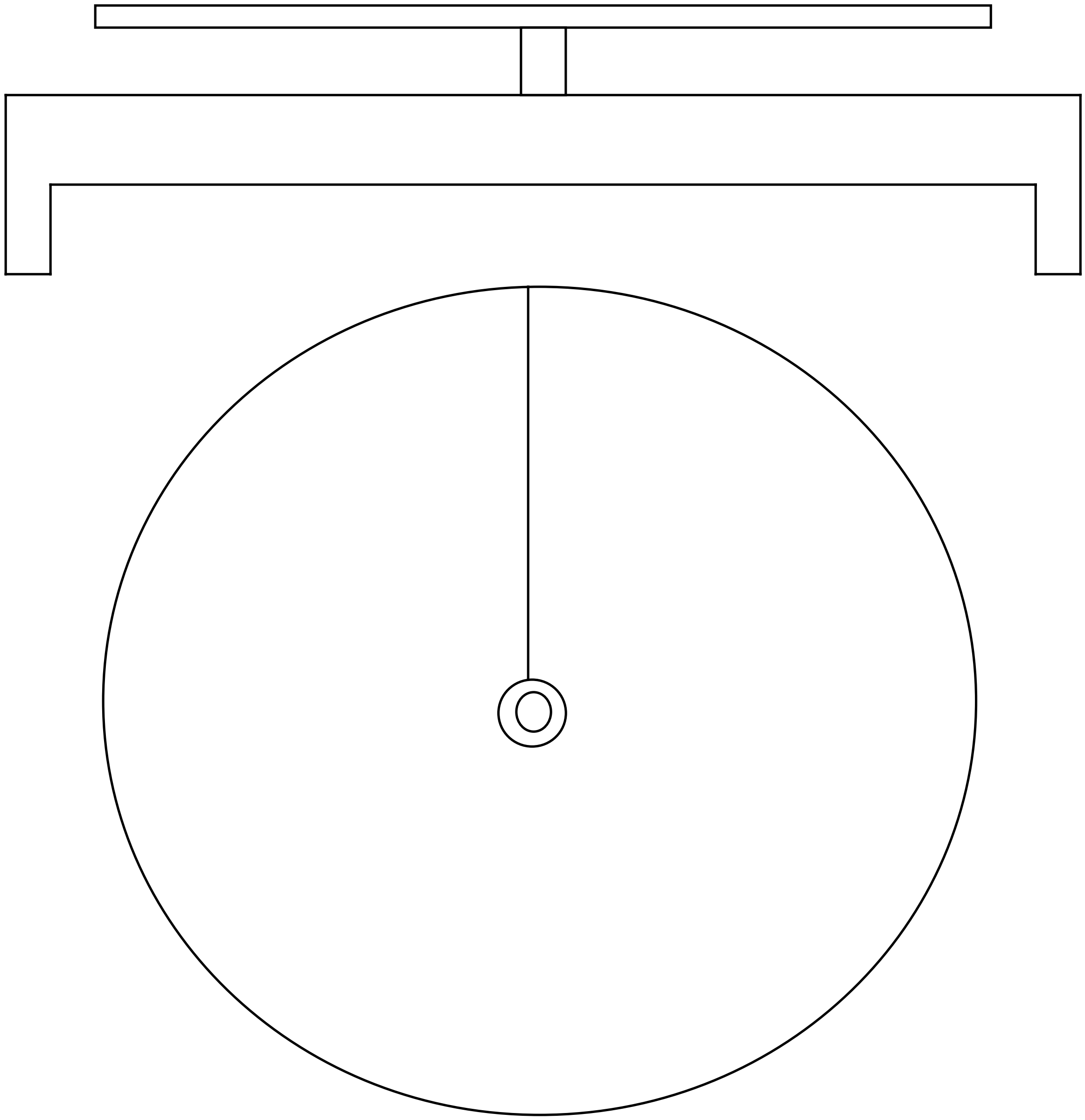
INDEPENDENCE

In probability theory, "independence" is not the negation of "dependence" We say that Y is " independent" of X only if knowing the value of X "doesn't change our uncertainty" about Y. More precisely, if we cripple our experiment $\mathcal{E}$ by any of the events $[X = a]$ the probabilities of all the events $[Y = b]$ do not change. Mathematically this is translated in the conditions that for all choices of a and b

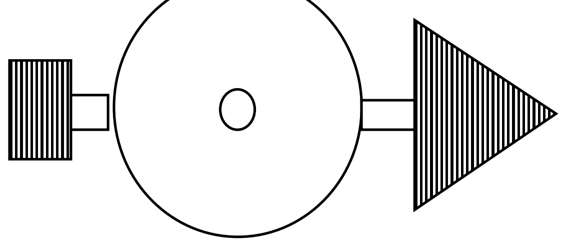
$$P[Y = b | X = a] = P[Y = b]$$

this simply means that

$$P[(Y = b) \cap (X = a)] = P[X = a] \times P[Y = b]$$



THE CANONICAL ROULETTE



COMPUTER SIMULATION

More can be found in D. KNUTH The art of Computer Programming II Chapter 3

STARTING SEQUENCE

$\boxed{Ro = SEED}$ (user chooses a 6-8 digit number)

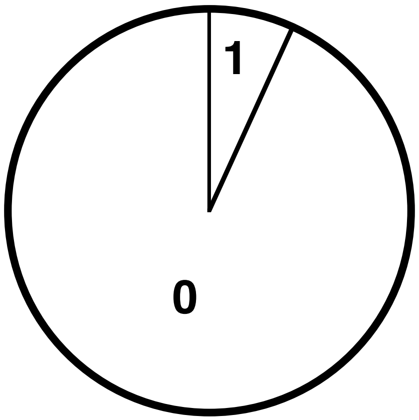
$$R_i = R_{i-1} \times 2^{27} \pmod{277998721}$$

for $i=1,2,...,30$

FOLLOWING SEQUENCE

$$R_k = R_{k-13} + R_{k-31} \pmod{277998721}$$

$$\omega_k = \frac{\mathbf{R}_k}{277998721}$$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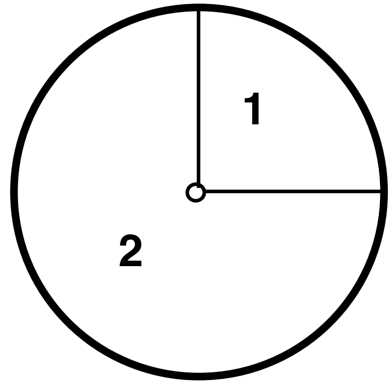
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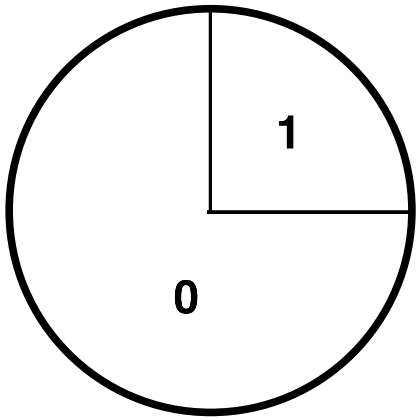
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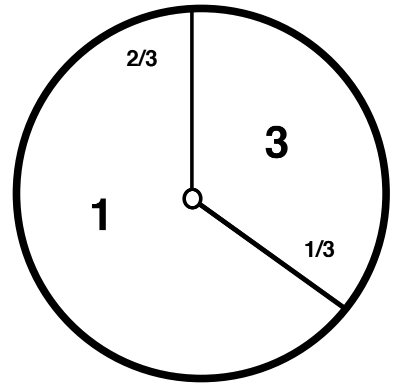
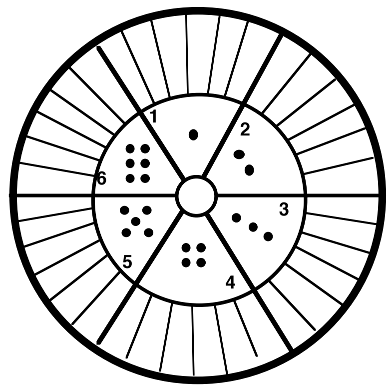

2/3
3
1
1/3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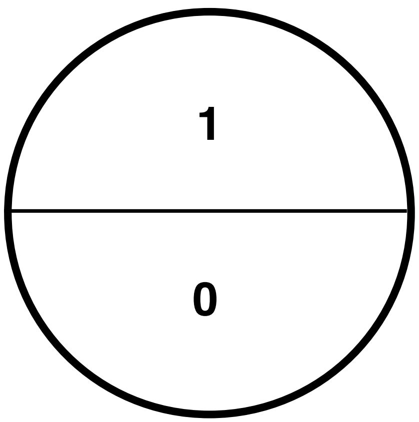
2
6
3
5
4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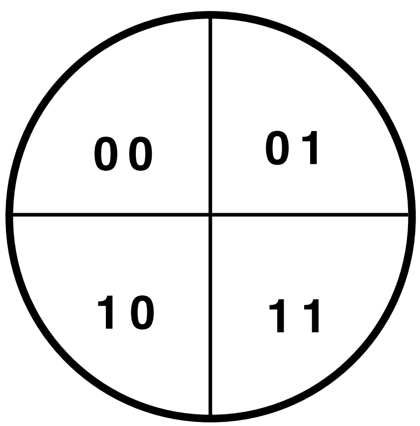
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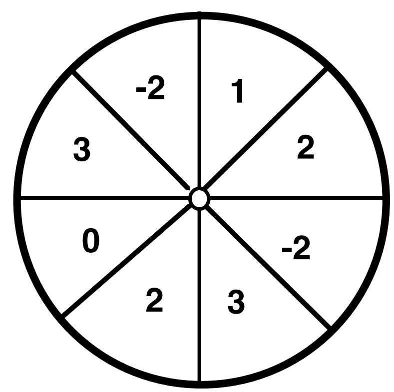
0 1
1 0
1 1

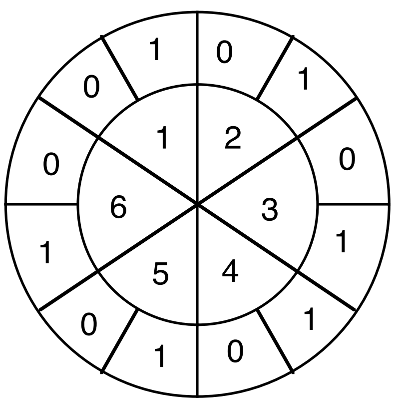

-2
1
3
2
0
-2
2
3

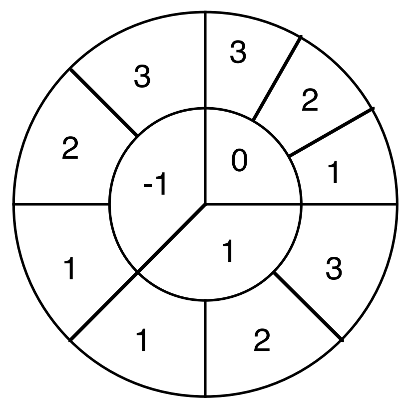
X is independent of Y if `P[X=a | Y = b] = P[X=a]`

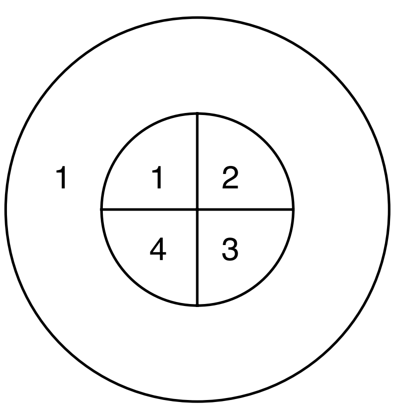
OR `P[X=a & Y=b] = P[X=a] P[Y=b]`

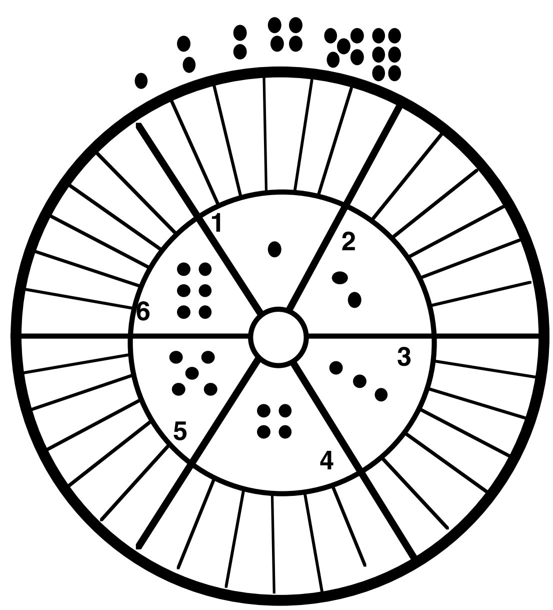
OR knowing the value of Y does not change the probabilities of X.

If X is independent of Y then Y is independent of X



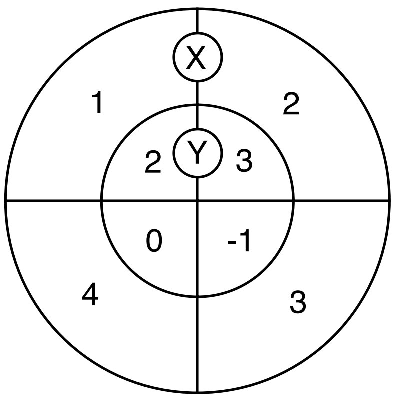


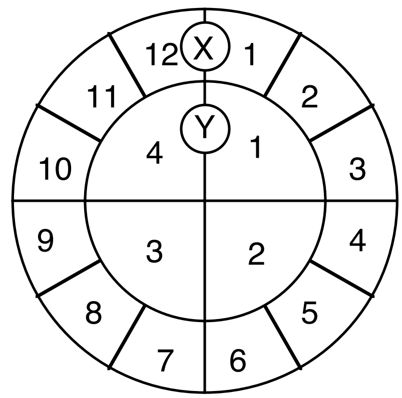


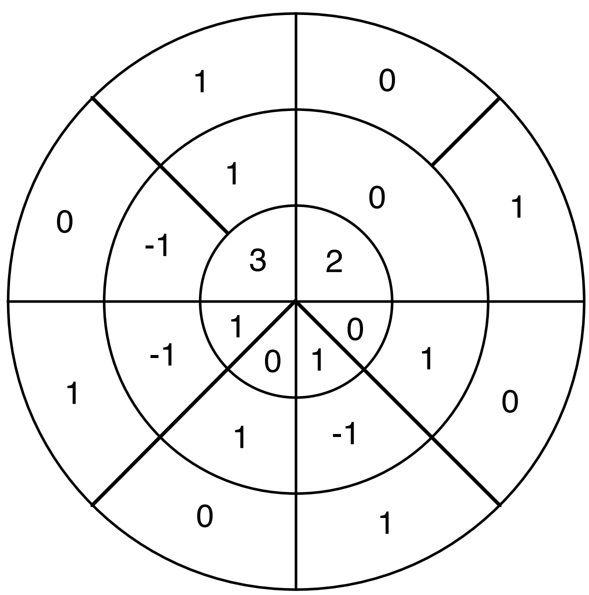


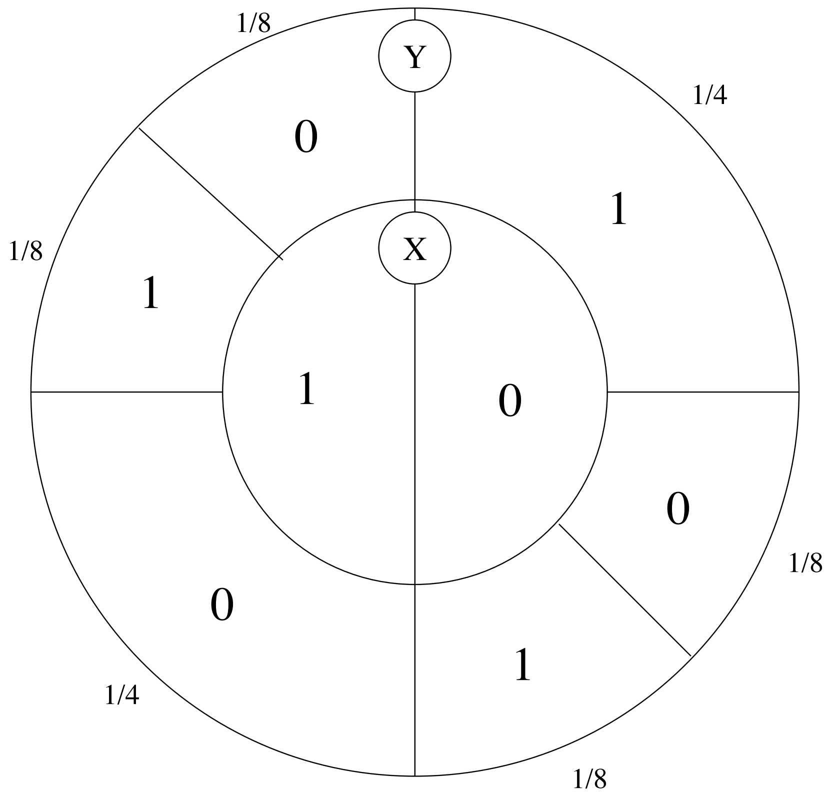
X is dependent on Y if X is a function of Y

knowing the value of Y determines the value of X.



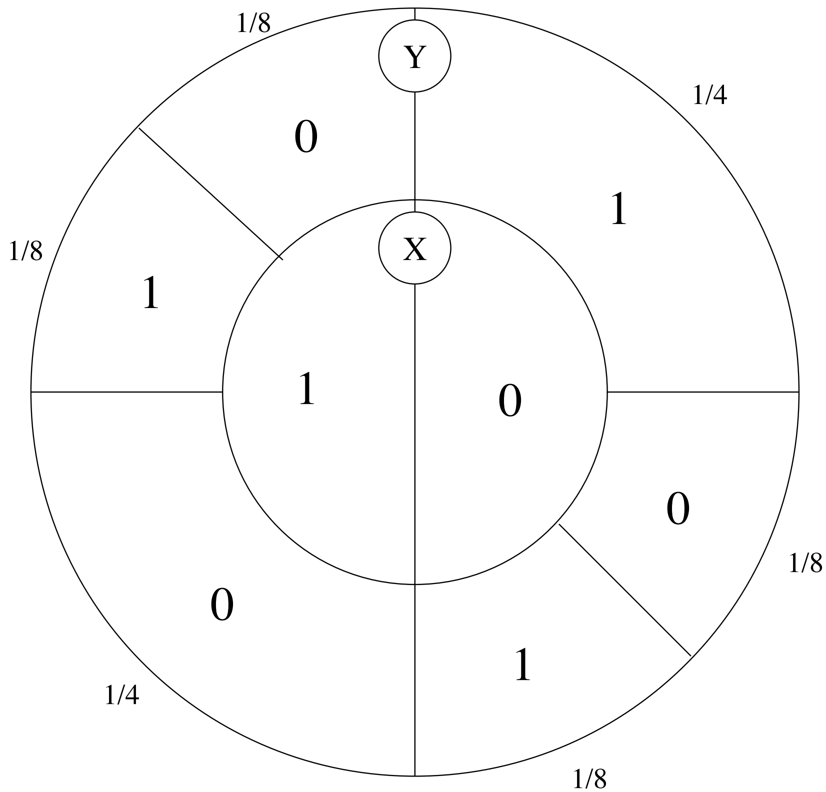




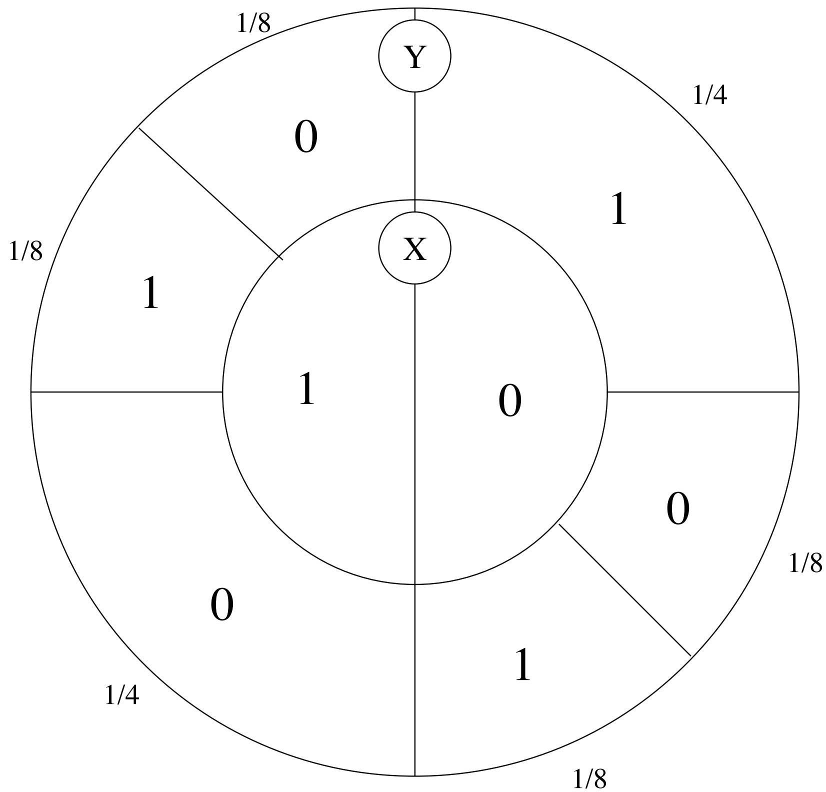


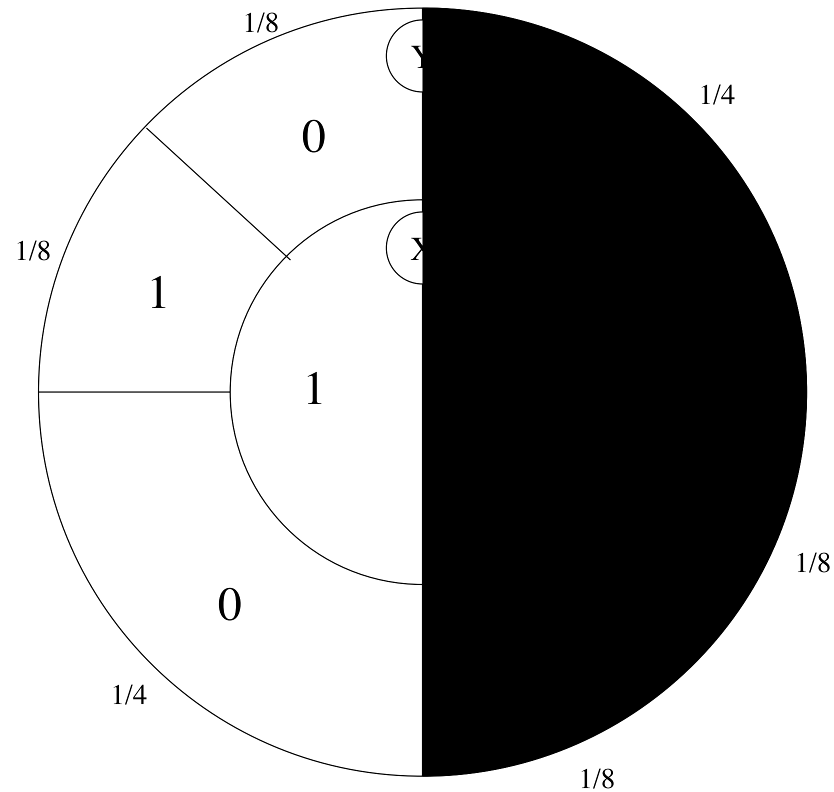
$P(X=1) = 1/2$ $P(X=0)=1/2$

$P(Y=1) = 1/2$ $P(Y=0)=1/2$

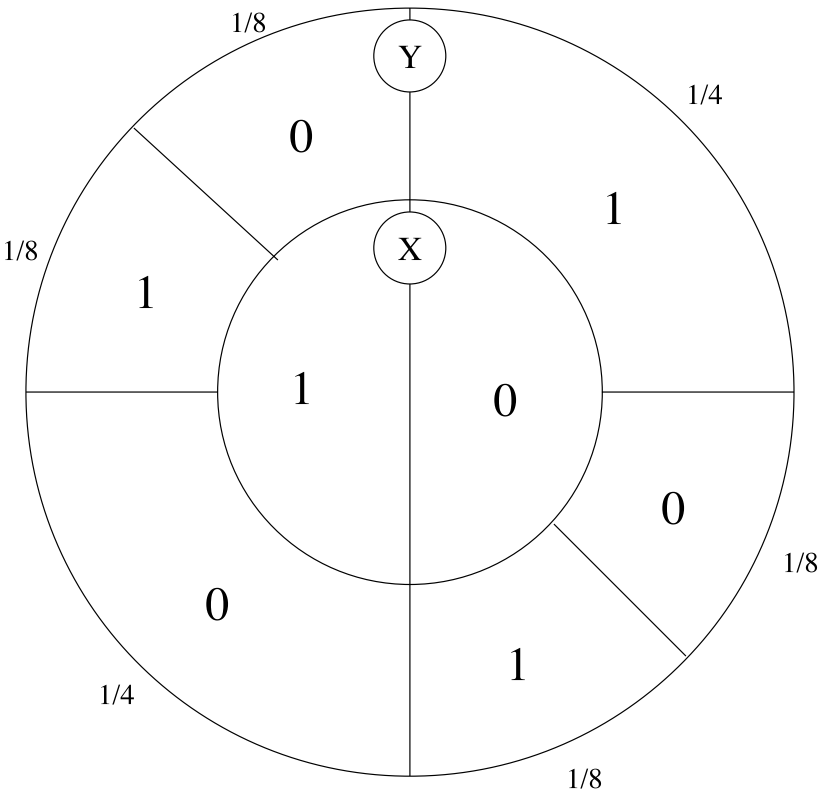


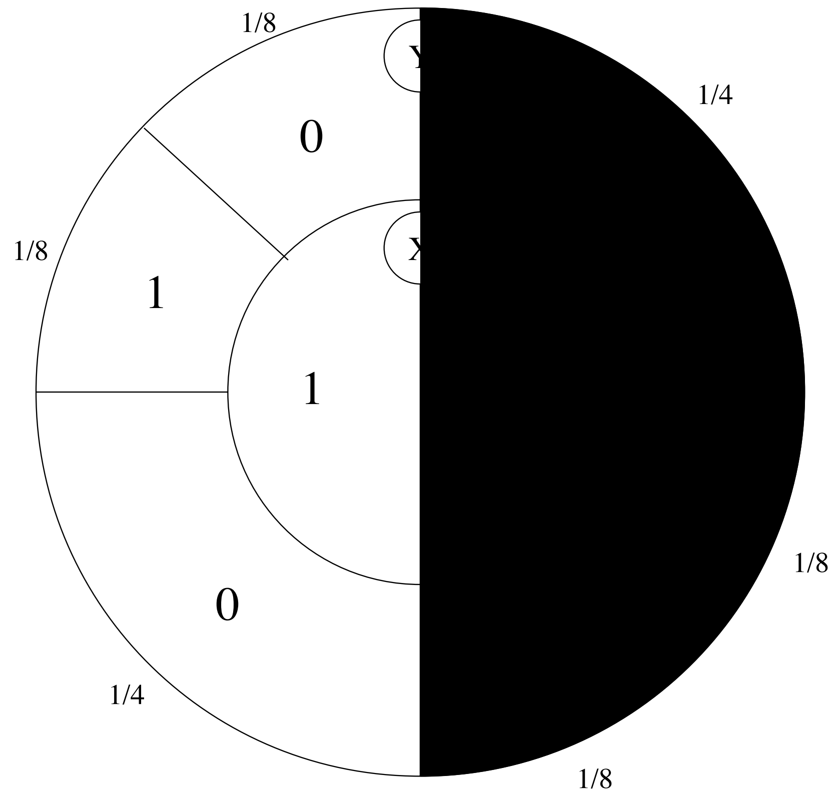
P(Y=0 | X=1) = ?



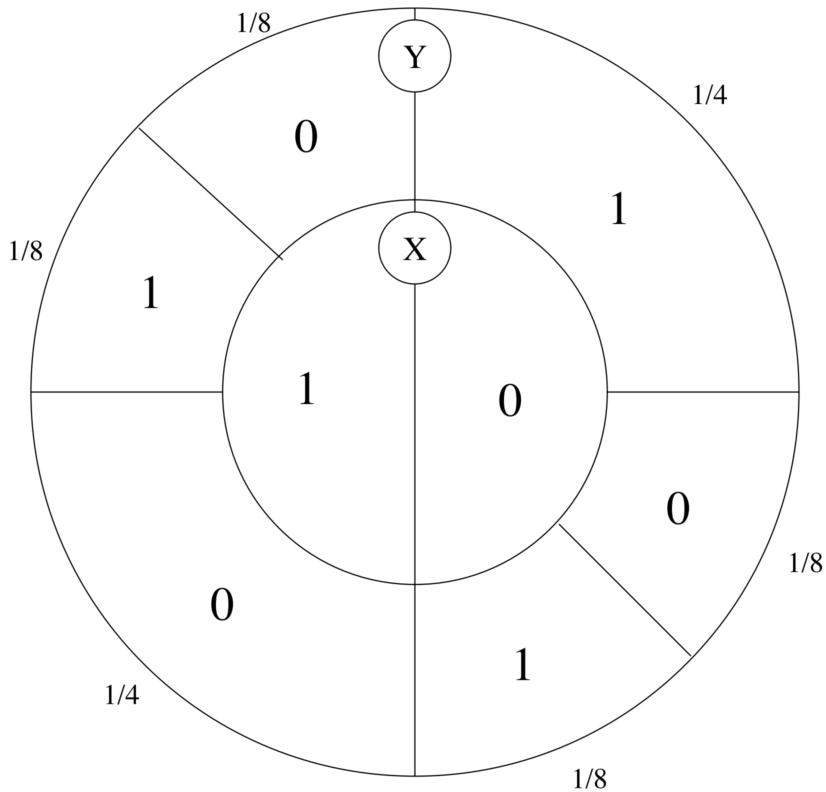


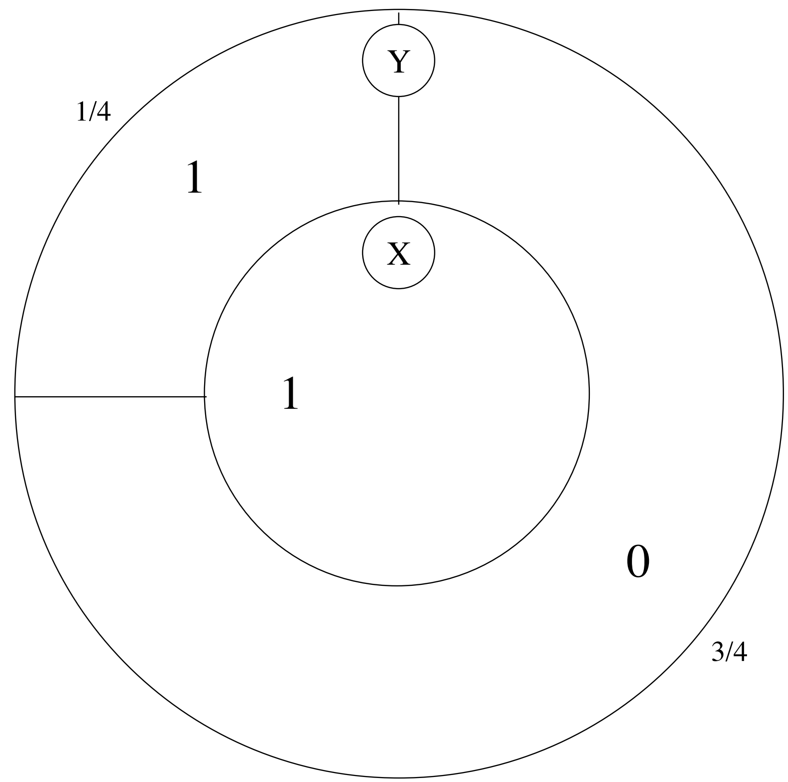
$$P(Y=0 \mid X=1) = ?$$





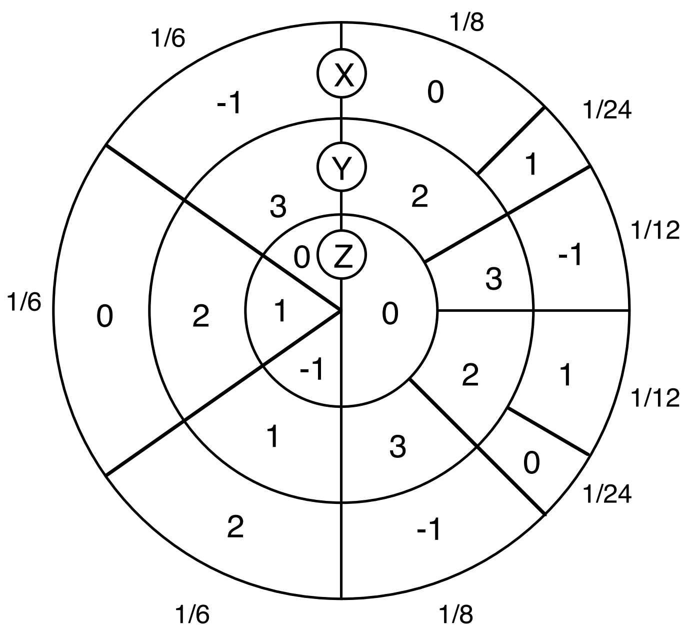
$$P(Y=0 \mid X=1) = \frac{P(Y=0 \text{ and } X=1)}{P(X=1)} = \frac{1/8+1/4}{1/2} = \frac{1/8+2/8}{1/2} = \frac{3}{4}$$





$$P(Y=0 \mid X=1) = \frac{P(Y=0 \text{ and } X=1)}{P(X=1)} = \frac{1/8+1/4}{1/2} = \frac{1/8+2/8}{1/2} = \frac{3}{4}$$

1. The wheel below represents the random variables X, Y and Z.



Calculate:

a) P(X=0)

b) P(X=1)

c) P(Z=-1 or X=0)

d) P(Y=2)

e) P(Y=2 or X=0)

f) P(Y=2 and X=0)

g) P(X=0 | Y=2)

h) P(X=0 | Z=-1)

i) P(X=2 | Z=-1)