MATH 2590 - ASSIGNMENT 3

OCTOBER 26, 2010

We will be using the concept of modular arithmetic again in this assignment, so if you need a definition or two, you might want to look back at the previous assignment. We will need an interesting fact. $a^p \equiv a \ (mod\ p)$ for all $1 \leq a < p$ (I will explain why this is true in class).

For the exercises below you will also need the following table:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Part I - Modular arithmetic

(1) Solve for a in the following equations
   (a) $5 \cdot a \equiv 1 \ (mod\ 26)$
   (b) $11 \cdot a \equiv 1 \ (mod\ 26)$
   (c) $21 \cdot a \equiv 1 \ (mod\ 26)$
   (d) $7 \cdot a \equiv 1 \ (mod\ 26)$

(2) Give a formula for y in terms of x in the following equations.
   (a) $11x + 20 \equiv y \ (mod\ 26)$
   (b) $21x + 6 \equiv y \ (mod\ 26)$
   (c) $7x + 16 \equiv y \ (mod\ 26)$

(3) Compute the following powers.
   (a) $11^{12} \ (mod\ 26)$
   (b) $21^{12} \ (mod\ 26)$
   (c) $7^{12} \ (mod\ 26)$

(4) Compute the following values of x.
   (a) $11^{5x} \equiv 11 \ (mod\ 26)$
   (b) $21^{7x} \equiv 21 \ (mod\ 26)$
   (c) $7^{5x} \equiv 7 \ (mod\ 26)$

(5) We know that all $1 \leq a \leq 112$, $a^{112} \equiv 1 \ (mod\ 113)$. Given this calculate x, such that
   (a) $11^{33x} \equiv 11 \ (mod\ 113)$
   (b) $21^{33x} \equiv 21 \ (mod\ 113)$
   (c) $7^{33x} \equiv 7 \ (mod\ 113)$
   (d) $a^{33x} \equiv a \ (mod\ 113)$

Date: due November 9, 2010.

Part II - Application of Encryption

(1) The following message is the name of an an animal and the letters were shifted by some amount. Figure out what A would be sent to under this transformation and find out what animal it was.

HCFHCWGS

(2) The following word was transformed by multiplying every letter by 5 (mod 26). Reverse the process by multiplying each corresponding letter by the value of a from problem (1)(a).

RUDUXJSNU

(3) Every letter in the message was transformed with multiplication by 11 followed by add 20 (mod 26). Reverse the process by using the equation for y you found in problem (2)(b).

FZUEH

(4) In the following message the numbers were found by taking each letter and raising it to the power of 33, (mod 113). Use what you learned from problem (5) to reverse this process.

83 - 7 - 87 - 15