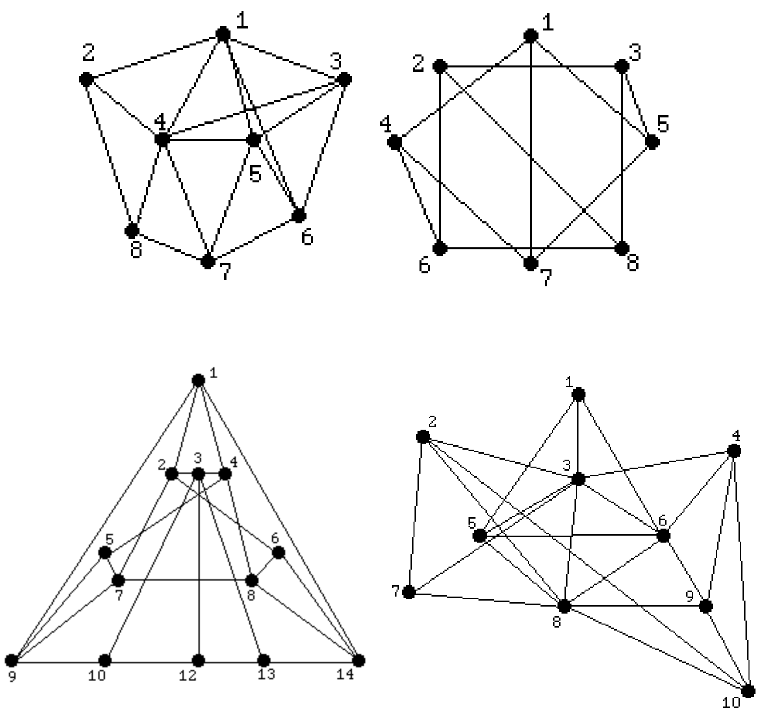
HOMEWORK #3 - MATH 3260

ASSIGNED: FEBRUARY 26, 2003 DUE: MARCH 12, 2003 AT 2:30PM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3) however you must register your group by the March 5, 2003 either by e-mail to `zabrocki@mathstat.yorku.ca` or in class with the signup sheet. You need only hand in one assignment per group, and write all names at the top.

(1) Show either that each of the following graphs are planar by drawing them in a way that the vertices do not cross or give a subgraph which is homeomorphic to $K_{3,3}$ or K_5. Hint: Two of these graphs are planar and two are non-planar.



(2) (a) Show that $K_{3,4}$ can be drawn in a torus without crossings by drawing this image. It will take some thought to determine how to arrange the vertices in a nice way.

(b) Corollary 14.3 on page 71 in the text gives a bound on the surface that a graph G can be embedded in. What does this say about the genus of the graphs $K_{r,s}$ for $r+s \geq 4$? What does this say in particular about $K_{3,3}$ and $K_{3,4}$?

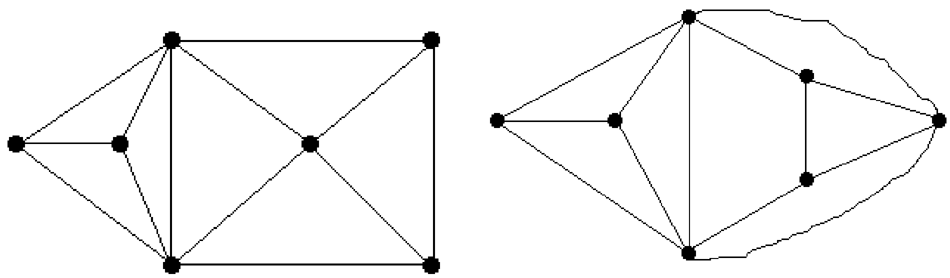
(c) Modify the proof of Corollary 14.3 to obtain a better lower bound for g if the graph does not contain any triangles. Show that this implies

$$g(K_{r,s}) \geq \left\lceil \frac{(r-2)(s-2)}{4} \right\rceil$$

In particular, explain in words what this inequality tells you about the graphs $K_{3,3}$ and $K_{3,4}$.

(3) Show that the dual of the cube graph is the octahedron graph and that the dual of the dodecahedron graph is the icosahedron graph. Give the number of vertices, edges and faces for each of these four graphs.

(4) Show that the following two graphs are isomorphic but that their geometric duals are not isomorphic.



(5) Let G be a simple planar graph. Demonstrate what conditions on G are necessary to guarantee that the dual of G has the following properties.

(a) G^* has no edges to and from the same vertex (no loops).

(b) G^{**} is isomorphic to G.

(c) G^* is bipartite.

(d) G^* is Eulerian.

Hint: These last two follow from Theorem 5.1 and Corollary 6.3 in the book.

(6) Find the chromatic number of

(a) each of the Platonic graphs.

(b) the complete graph K_n for $n \geq 1$

(c) the complete bipartite graph $K_{r,s}$ for $r, s \geq 1$.

(d) the complete tripartite graph $K_{r,s,t}$ for $r, s, t \geq 1$.

(e) the k-cube Q_k for $k \geq 1$.