HOMEWORK #4 - MATH 3260

ASSIGNED: MARCH 19, 2003 DUE: APRIL 4, 2003 AT 2:30PM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3) however you must register your group by the March 5, 2003 either by e-mail to `zabrocki@mathstat.yorku.ca` or in class with the signup sheet. You need only hand in one assignment per group, and write all names at the top.

(1) (a) Prove that the chromatic polynomial of any tree with s vertices is

$$k(k-1)^{s-1}$$

(b) Prove that the chromatic polynomial of $K_{2,s}$ is

$$k(k-1)^s + k(k-1)(k-2)^s$$

(c) Prove that the chromatic polynomial of C_n is

$$(k-1)^n + (-1)^n(k-1)$$

(2) Let G be a simple graph with n vertices and m edges. Use induction on m, together with Theorem 21.1, to prove that
(a) the coefficient of k^{n-1} is $-m$
(b) the coefficients of $P_G(k)$ alternate in sign.

(3) Prove that, if $G = G(V_1, V_2)$ is a bipartite graph in which the degree of every vertex in V_1 is not less than the degree of each vertex in V_2 then G has a complete matching.

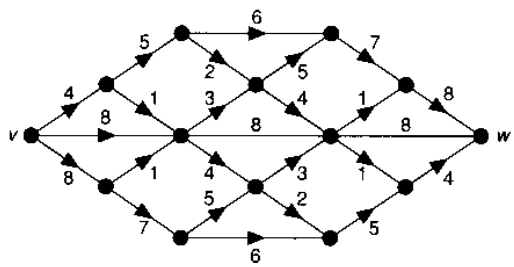
(4) A schedule for finals is to be drawn up for a group of 7 classes, a through g. Two classes may not be scheduled at the same time if there exists a student in both classes. The table below shows the classes which may not be scheduled at the same time (marked with a $\bullet$). What is the minimum number of time slots are needed to schedule all 7 classes?

	a	b	c	d	e	f	g
a		$\bullet$	$\bullet$	$\bullet$			$\bullet$
b	$\bullet$		$\bullet$	$\bullet$	$\bullet$		$\bullet$
c	$\bullet$	$\bullet$		$\bullet$		$\bullet$	
d	$\bullet$	$\bullet$	$\bullet$			$\bullet$	
e		$\bullet$					
f			$\bullet$	$\bullet$			$\bullet$
g	$\bullet$	$\bullet$				$\bullet$	

(5) Verify the statements of Corollary 26.2 when $E = \{a, b, c, d, e\}$ and $\mathcal{F} = (\ \{a, c, e\}, \{b, d\}, \{b, d\}, \{b, d\})$. This means that you must find a t such that a partial transversal of size t exists and verify for each $1 \leq k \leq 4$ that every subcollection of subsets contains at least $k + t - m$ elements.

(6) Let E be the set $\{1, 2, \ldots, 6\}$. How many transversals do the following families have? Justify your answer.
  (a) $(\{1,2,3\}, \{2,3,4\}, \{3,4,5\}, \{4,5,6\}, \{5,6,1\})$
  (b) $(\{1\}, \{2,3\}, \{1,2\}, \{1,3\}, \{1,4,6\})$
  (c) $(\{1,2\}, \{2,3\}, \{2,4\}, \{2,5\}, \{2,6\})$
  (d) $(\{1,3,5\}, \{2,4\})$
  (e) $(\{1,3,5\}, \{2,3,4\})$

(7) Find a flow with value 20 in the network in the following figure. Is it a maximum flow?



(8) List all of the cuts in the following network and find a minimum cut. Find a maximum flow and verify that this satisfies the max-flow min-cut theorem.