MIDTERM - TAKE HOME - MATH 4160

ASSIGNED: OCTOBER 11, 2012   DUE: OCTOBER 16, 2012 AT 2:30PM

Write your solutions neatly and clearly. Provide full explanations and justify all of your answers.
DO NOT DISCUSS THESE PROBLEMS WITH OTHERS. You must do this work alone and I
will ask you to sign the statement below which states that you have not discussed these problems
with others or received help on these problems (when you hand the paper to me). Note that in
certain circumstances I am giving you the answer, and it is your job *explain* it. This means that
you should write grammatically correct sentences, tell me why two things are equal, and make your
calculations clear and easy to follow.

If you have any questions about the problems you may e-mail me at
`zabrocki@mathstat.yorku.ca`.

1 (4pts)	.
2 (6pts)	
3 (4pts)	
4 (6pts)	
total(20pts)	

(1) (4 pts) Show that the sequence satisfying $d_0 = 1$ and $d_1 = 2$ and $d_n = 3d_{n-1} - d_{n-2} + 1$ for
$n \geq 2$ has the generating function

$$D(x) = \sum_{n \geq 0} d_n x^n = \frac{1 - 2x + 2x^2}{(1 - x)(1 - 3x + x^2)}.$$

Use this to explain and find an expression for the generating function

$$E(x) = \sum_{n \geq 0} (nd_n - 1) x^n$$

Hint: $E(x) = \frac{-1 + 9x - 21x^2 + 20x^3 - 11x^4 + 3x^5}{(1-x)^2(1-3x+x^2)^2}.$

(2) (6 pts) For the following identity give a combinatorial proof by describing a set that is
counted by the left hand side of the equality and a set that is counted by the right hand
side of the equality and explaining why these two sets are the same size.

$$\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \cdots + \binom{n}{3} = \binom{n+1}{4}.$$

(3) (4 pts) Give an expression for the generating function for the number of non-negative integer
solutions to

$$x_1 + x_2 + 2x_3 + 3x_4 = n$$

with $x_1 + x_2 \geq 8$.

(4) (6 pts) Explain why the number of 3 digit numbers such that the digit 0 appears zero times,
the digit 1 appears at most one time, the digit 2 appears at most two times is equal to 703.

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WHEN YOU SUBMIT THIS EXAM please sign the following statement and fill out the information below.

I attest that I have completed this exam myself without help from anyone else and I have not discussed the problems on this exam with other students in the class.

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This exam is open book, open notes, and other sources, but I expect you to not ask other people how to complete the assignment. Everyone should list books and websites that you consulted below. If you cannot sign the above statement truthfully, I would prefer if you just explain to me the situation rather than perjure yourself. Please detail below the sources you consulted, the that you have obtained on this exam or who you have discussed these problems with: