PARTITION GENERATING FUNCTIONS

A Durfee square is the largest square which can be fit inside of the diagram of a partition.

Apply the addition or the multiplication principle of generating functions to give the generating function for the following sequences of numbers.

(1) the number of partitions of n with parts that are multiples of 3.
(2) the number of partitions of n with only even parts and all parts not occurring more than twice.
(3) the number of partitions of n with distinct parts which are all congruent to 1 or 4 (mod 5).
(4) the number of partitions of n with parts of size 1, 2 or 3 occurring at most 8 times each.
(5) the number of partitions of n with at most 8 parts of any given size.
(6) the number of partitions of n with even parts and 0, 1 or 2 parts of any given size.
(7) the number of partitions of n with a Durfee square of odd size.
(8) the number of partitions of n with distinct parts and no parts are equivalent to 3 (mod 5).
(9) the number of partitions of n with parts of size less than or equal to k and each part occurs at most 4 times.
(10) the number of partitions of n with with odd parts and a part will either occur 0 or an odd number of times.
(11) the number of partitions of n into parts congruent to 2 or 3 (mod 5).
(12) the number of partitions of n with length less than or equal to 5 and a Durfee square of size 1×1, 3×3 or 5×5.
(13) the number of partitions of n with a Durfee square of odd size and all parts even.
(14) the number of partitions of n with Durfee square equal to 3 and all odd parts.
(15) the number of partitions of n with length exactly equal to 4.
(16) the number of partitions of n with less than or equal to k parts and each part occurs an even number of times.
(17) the number of partitions of n with no parts that are equivalent to 3 (mod 5).
(18) the number of partitions of n with a Durfee square of size 1×1, 2×2 or 3×3.
(19) the number of partitions of n with Durfee square of size 4×4 and all distinct parts.
(20) the number of partitions of n with distinct parts and all parts are equivalent to 3 (mod 5).
(21) the number of partitions of n with length exactly equal to 5 and all parts distinct.
(22) the number of partitions of n with even parts and all parts occurring an even number of times.
(23) the number of partitions of n with odd parts and all parts occurring an even number of times.
(24) the number of partitions of n with a Durfee square of size 3×3 and the length of the partition less than or equal to 4.
(25) the number of partitions of n with distinct parts, a Durfee square of size 3×3 and length equal to 3.
(26) the number of partitions of n with less than or 4 parts and all parts distinct.
(27) the number of partitions of n with even parts occurring an even number of times and odd parts occurring any number of times.
(28) the number of partitions of n with Durfee square of size 3×3 and all even parts.
(29) the number of partitions of n with length less than or equal to 3 and all parts distinct.
(30) the number of partitions of n with odd parts and each part occurs at most 4 times.
(31) the number of partitions of n with length exactly equal to 4 and all parts distinct.
(32) the number of partitions of n with a Durfee square of even size and all parts even.