SOME PROBLEMS FOR NEXT TIME

ASSIGNED: SEPT 27, 2012, WILL DO IN CLASS OCT 2, 2012

If $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ is the generating function for the sequence $a_0, a_1, a_2, a_3, \ldots$, and $g(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots$ is the generating function for the sequence $b_0, b_1, b_2, b_3, \ldots$, then what is the generating function for

(1) $a_0, a_3, a_6, a_9, \dots$ (2) $0a_0, 1a_1, 2a_2, 3a_3, \dots$ (3) $\binom{0}{k} a_0, \binom{1}{k} a_1, \binom{2}{k} a_2, \binom{3}{k} a_3, \dots$

What is the sequence with generating function given by

(1) $f(x)^2$ (2) f(x)g(x)(3) f(x)/(1-x)(4) f(g(x))

Note that the last one is on the hard side until we come up with clever notation about how to express this clearly, but I am suggesting it because you might be able to figure out what the nice notation is if you give it enough thought.