

MIDTERM # 1 - MATH 4160 - JANUARY 31, 2003

YOU MAY USE YOUR BOOK, NOTES OR CALCULATOR

- (1) (20 points) How many 10 card hands from a deck of 52 have two straight flushes of 5 cards each consisting of two different suits? (e.g. $3\spadesuit 4\spadesuit 5\spadesuit 6\spadesuit 7\spadesuit 6\heartsuit 7\heartsuit 8\heartsuit 9\heartsuit 10\heartsuit$ would be such a hand. The ace may be a high or a low card but not both).

- (2) (a) (10 points) How many ways are there of rearranging the letters of the word UNDERHANDED?
- (b) (10 points) How many ways are there of rearranging the letters so that there are no two Ds next to each other?

- (3) In the California lottery there is a game called SuperLotto PlusSM where a winning ticket has 5 numbers correct from 47 plus a Mega Number that ranges from 1 to 27 (and may be the same as the other 5 numbers).
- (a) (10 points) The odds for matching 4 of the 5 numbers plus the Mega Number is 1 in 197,221. Explain this probability.
 - (b) (10 points) The odds of matching 5 out of the 5 numbers and not the Mega Number is 1 in 1,592,937. Explain this probability.

- (4) (20 points) Prove one of the following identities by giving a combinatorial interpretation to both sides of the equation and explaining why they must be equal:

$$\binom{m+n}{m+r} = \sum_{k=0}^m \binom{m}{m-k} \binom{n}{r+k}$$
$$\binom{n}{k} \binom{k}{\ell} = \binom{n-1}{k} \binom{k}{\ell} + \binom{n-1}{k-1} \binom{k-1}{\ell} + \binom{n-1}{k-1} \binom{k-1}{\ell-1}$$

- (5) (20 points) Use inclusion-exclusion to solve the following question. Clearly write out descriptions of sets whose use will appear in the application of the inclusion-exclusion principle:

How many 4 digit numbers are there (consisting of the digits 0 through 9) with no digit appearing exactly two times.