TAKE HOME ALTERNATE EXAM - MATH 4160

ASSIGNED: FEBRUARY 5, 2003 DUE: FEBRUARY 7, 2002 AT 2:30PM

Write your solutions neatly and clearly. Provide full explanations and justify all of your answers. DO NOT DISCUSS THESE PROBLEMS WITH OTHER STUDENTS. You must do this work alone and I will ask you to sign the statement below which states that you have not discussed these problems with others or received help on these problems (when you hand the paper to me).

If you have any questions about the problems you may e-mail me at `zabrocki@mathstat.yorku.ca`.

(1) Consider a 10 card hand from a deck of 52 cards. How many of these hands contain a three-of-a-kind and a straight of 7 cards (both disjoint) such that the hand does not have a four-of-a-kind. (e.g. If the 3-of-a-kind is $8\heartsuit, 8\spadesuit, 8\diamondsuit$ then the 7 card straight must have the cards A through 7 because the straight cannot contain an 8 so a valid hand will be $8\heartsuit, 8\spadesuit, 8\diamondsuit, A\heartsuit, 2\clubsuit, 3\clubsuit, 4\spadesuit, 5\diamondsuit, 6\clubsuit, 7\spadesuit$). The ace is high or low but not both.

(2) How many rearrangements are there of the letters of the words LIES AND STATISTICS? Do not ignore spaces and count rearrangements of these letters so that there are three distinct words (each must have at least one letter).

(3) The California lottery has a game called 'Hot Spot' where a player picks 8 numbers from 1 to 80 and the every 5 minutes 20 new winning numbers are chosen. The player wins if they match either 0,5,6,7 or 8 of their numbers against the winners.
   (a) The odds of winning the 0 of 8 prize is approximately 1 in 11. Explain this probability.
   (b) The odds of matching 7 of 8 numbers is approximately 1 in 6232. Explain this probability.

(4) Prove one of the following identities by giving a combinatorial interpretation to both sides of the equation and explain why they must be equal:

$$\binom{n}{k+\ell}\binom{k+\ell}{\ell} = \binom{n-k}{\ell}\sum_{r=0}^{k}\binom{k}{r}\binom{n-k}{k-r}$$

OR

$$\binom{m+n+1}{r+s+1} = \sum_{k=0}^{m}\binom{m-k}{r}\binom{n+k}{s}$$

(5) Use inclusion-exclusion to solve the following question. Clearly write out descriptions of sets whose use will appear in the application of the inclusion-exclusion principle:
   How many 8 letter sequences are there with at least one letter appearing exactly twice.

WHEN YOU SUBMIT THIS EXAM please sign the following statement.

I attest that I have completed this exam myself without help from anyone else and I have not discussed the problems on this exam with other students in the class.