

## MIDTERM # 2 - TAKE HOME - MATH 4160

ASSIGNED: MARCH 7, 2003

DUE: MARCH 10, 2003 AT 10:30AM

Write your solutions neatly and clearly. Provide full explanations and justify all of your answers. DO NOT DISCUSS THESE PROBLEMS WITH OTHER STUDENTS. You must do this work alone and I will ask you to sign the statement below which states that you have not discussed these problems with others or received help on these problems (when you hand the paper to me).

If you have any questions about the problems you may e-mail me at [zabrocki@mathstat.yorku.ca](mailto:zabrocki@mathstat.yorku.ca).

- (1) Say that there are exactly 100 unlabeled balls in a bin but that they are colored so that 25 are red, 25 are green, 25 are blue and 25 are yellow. Reach in the bin and pick out 4 balls. What is the probability of choosing them such that 2 balls are one color and 2 balls are a second color?
- (2) Consider paths in a 3-dimensional grid starting at the origin  $(0, 0, 0)$  and ending at the point  $(n, k, \ell)$ .
  - (a) How many paths are there if there are only steps in the  $(+1, 0, 0)$ ,  $(0, +1, 0)$ , or  $(0, 0, +1)$  direction?
  - (b) Write down a formula for the number of paths there are if in addition to steps in the  $(+1, 0, 0)$ ,  $(0, +1, 0)$  or  $(0, 0, +1)$  directions that there are also steps in the  $(+1, +1, +1)$  direction. Hint: grade your answer by the number of diagonal steps.
- (3) Give a formula for the generating function of the number of partitions satisfying there is exactly one even part and every odd part is not repeated more than twice (e.g.  $(4, 3, 1, 1)$  is a partition of this type but  $(4, 3, 1, 1, 1)$  is not).
- (4) Say that  $a_i$  is the number of *widgits* of size  $i$  and let  $A(x) = \sum_{n \geq 0} a_n x^n$  be the generating function for these numbers. What is the coefficient of  $x^n$  in the expression  $(1+x)^m A(x) = \sum_{k=0}^m \binom{m}{k} x^k A(x)$ ? Give an explanation of the meaning of this coefficient in terms of *widgits*.
- (5) Find a formula for the generating function  $D(x) = \sum_{n \geq 0} d_n x^n$  for the sequence  $d_n$  that satisfies the recurrence  $d_n = 2d_{n-1} - d_{n-2} + n$  and  $d_0 = 1$  and  $d_1 = 3$ .

WHEN YOU SUBMIT THIS EXAM please sign the following statement.

I attest that I have completed this exam myself without help from anyone else and I have not discussed the problems on this exam with other students in the class.

If you cannot sign the above statement truthfully, please detail below the help that you have obtained on this exam or who you have discussed these problems with: