HOMEWORK #1 - MATH 4160

ASSIGNED: JAN 13/03, REVISED: JAN 16/03, DUE: JAN 24/03 AT 10:30AM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3) however you must register your group by the January 17, 2002 either by e-mail to `zabrocki@mathstat.yorku.ca` or in class with the signup sheet. You need only hand in one assignment per group, and write all names at the top.

Note there are hints and clearer explanations for these homework problems on the web page:
`http://garsia.math.yorku.ca/~zabrocki/math4160w03/hintshw1/`

Enumeration problems:

(1) How many ways are there of placing two Kings on an 8×8 chessboard so that they are not on adjacent squares?
(2) How many ways are there of placing two Queens on an 8×8 chessboard so that the pieces are not on the same row, column or diagonal?
(3) How many different r^{th} order partial derivatives does $f(x_1, x_2, \ldots, x_n)$.
(4) How many arrangements are there of five as, five bs and five cs with at least one b and at least one c between each successive pairs of a?
(5) How many bridge hands are there with the suit distribution $6-3-2-2$?
(6) In a bridge deal you and your partner between you have 9 spades. What is the probability that one of your two opponents has exactly three of the four remaining spades?

Combinatorial proofs: For the following problems give a combinatorial proof by describing a set that is counted by the left hand side of the equality and a set that is counted by the right hand side of the equality and explaining why these two sets are the same.

(1)
$$\sum_{r=0}^{n} \binom{n}{r}\binom{k}{r} = \binom{n+k}{n}$$

(2)
$$\binom{n-1}{0} + \binom{n}{1} + \binom{n+1}{2} + \cdots + \binom{2n-1}{n} = \binom{2n}{n}$$

(3)
$$\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \cdots + \binom{n}{r} = \binom{n+1}{r+1}$$

(4)
$$\binom{n}{k}\binom{k}{\ell} = \binom{n}{\ell}\binom{n-\ell}{n-k}$$