HOMEWORK #2 - MATH 4160

DUE: WEDNESDAY FEBRUARY 5, 2002 AT 10:30AM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3). You need only hand in one assignment per group, and write all names at the top.

Enumeration problems. Do problem #1 and two of #2, #3 or #4:

(1) How many arrangements are there of the letters in the words ELEMENTARY ENUMERATION such that there are no consecutive E's?
(2) How many quick pick outcomes are there in the *Super 7* lottery game (pick 7 numbers in 49)? Say 7 winning numbers are fixed and a bonus number is chosen. How many of these quick pick numbers have all 7 numbers correct? 6 numbers and a bonus? 6 numbers and not the bonus? Five numbers?
(3) The following questions are regarding a poker hand of five cards.
   (a) How many hands contain a pair and three more cards that have the same suit as one of the cards in the pair? (e.g. $3\heartsuit, 3\clubsuit, 6\clubsuit, 8\clubsuit, Q\clubsuit$)
   (b) How many hands contain a pair and three more cards in sequence of the same suit (which may or may not be the same as the suit in the pair)? (e.g. $5\heartsuit, 5\spadesuit, J\clubsuit, Q\clubsuit, K\clubsuit$ or $7\heartsuit, 7\diamondsuit, 2\diamondsuit, 3\diamondsuit, 4\diamondsuit$) The sequence of 3 should not share a card with the pair or even have a card in common with the pair (e.g. the hand $5\heartsuit, 6\heartsuit, 7\heartsuit, 6\clubsuit, 6\diamondsuit$ is not valid).
(4) There are 100 pennies, how many ways are there of distributing them to 5 different people (i.e. each person may or may not get some of these pennies)? How many ways are there of distributing these pennies so that each person gets at least 5 pennies each? (Hint: think of the derivative problem from last time).

Do three of the following 6 problems. These will use the principle of inclusion-exclusion.

(1) How many n-digit decimal sequences (using the digits $0-9$) are there in which the digits $1, 2$ and 3 all appear?
(2) How many ways are there of rolling a sided die 10 times in a sequence such that all 6 faces appear at least once?
(3) How many positive integers ≤ 420 are relatively prime to $420 = 5 \cdot 2^2 \cdot 7 \cdot 3$?
(4) How many arrangements of 52 letters, 2 As, 2 Bs, 2 Cs, etc. with no pair of consecutive letters the same?
(5) How many ways are there of dealing a 13 card hand with at least suit that does not appear?
(6) How many 13 card hands have at least one type of face card (J, Q, K, A)?