

HOMEWORK #3 - MATH 4160

DUE: FRIDAY FEBRUARY 14, 2002 AT 10:30AM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3). You need only hand in one assignment per group, and write all names at the top.

Enumeration problems.

- (1) How many paths are there that start at the point $(0, 0)$ and go to the point $(2n, 0)$ by taking steps in a NE or SE direction (i.e. $(+1, +1)$ or $(+1, -1)$)?
- (2) Bonus: How many of these paths go below the line $y = 0$?

Generating functions:

- (1) Let $p_{\leq k}(n)$ represent the number of partitions of n with width less than or equal to k . Let $P_{\leq k}(x) = \sum_{n \geq 0} p_{\leq k}(n)x^n$ be the generating function for this sequence. Prove by induction on k that generating function for this sequence is

$$P_{\leq k}(x) = \prod_{r=1}^k \frac{1}{1-x^r}.$$

- (2) Define the Fibonacci numbers as the sequence satisfying $f_0 = 1$, $f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n > 1$. Find an algebraic expression for the generating function $F(x) = \sum_{n \geq 0} f_n x^n$.