MORE COUNTING PROBLEMS

PROBLEM SESSION #2 - JANUARY 27, 2003

(1) How many 7 card hands from a deck of 52 have:
    (a) a three-of-a-kind and two two-of-a-kinds? (e.g. $J\spadesuit, J\heartsuit, J\clubsuit, Q\diamondsuit, Q\heartsuit, 2\clubsuit, 2\heartsuit$)
    (b) a four-of-a-kind and 3 other cards which are in sequence?(e.g. $5\spadesuit, 5\heartsuit, 5\clubsuit, 5\diamondsuit, 7\diamondsuit, 8\heartsuit, 9\diamondsuit$)
    (c) a four-of-a-kind and a 3 cards of the same suit? (e.g. $K\spadesuit, K\heartsuit, K\clubsuit, K\diamondsuit, 3\heartsuit, 8\heartsuit, J\heartsuit$)
(2) How many sequences of letters consisting of 5 As, 5 Bs and 5 Cs are there that
    (a) begin with 3 As?
    (b) begin with ABC?
    (c) every A has a B immediately to the right?
    (d) No two Bs are adjacent?
    (e) (*) No two As are adjacent, and no two Bs are adjacent?
    (f) (*) No two As are adjacent, and no two Bs are adjacent and no two Cs are adjacent?
(3) How many ways are there of rolling a 6-sided die 6 times in a row such that
    (a) there are exactly four 3s in a row somewhere?
    (b) there are at least four 3s in a row?
    (c) there are more 3s than 4s?
    (d) no 3s appear in the sequence?
(4) In the Super 7 lottery, 7 numbers are chosen from the numbers 1 through 47, and it is more difficult to win than the 6/49 game so much so that they give you three chances to win on the same ticket. If you did a quick pick they give you 3 sets of 7 numbers but in theory it is possible that two of the three sequences are the same. What is the probability that this happens?
(5) Prove the following identity by giving a combinatorial interpretation to both sides of the equation and explain why they must be equal:
$$\binom{2n}{n} = \sum_{i=0}^{n}(n-i+1)\binom{n+i-2}{i}$$
(6) Use inclusion-exclusion to solve the following question:
    How many sequences of the five numbers 1 through 5 with no numbers repeated have at least one number i appearing in the ith position?

(*) Might be hard.