Entropy of random letters with probability of each letter chosen 1/26

$$\sum_{lpha=A}^Z 1/26 \log_2(26) pprox 4.7$$

• Entropy of letters in English chosen independently using single letter probabilities  $p_{\alpha}$ 

$$\sum_{lpha=A}^Z p_lpha \log_2(1/p_lpha) pprox 4.16$$

- Entropy of English using biletter statistics  $\approx 3.2$
- Experimental entropy of English with at least 25 letters pprox 1.2

Let F represent the of number of bits of information per English letter of text.

$$\begin{array}{ll} n < 8 & F = 4.16 \\ 8 < n \leq 15 & F = 3.2 \\ 15 < n \leq 25 & F = 2 \\ 25 < n & F = 1.2 \end{array}$$

For a cyphertext only attack to estimate the unicity distance (set H(K|C) = 0) we use the equation

$$H(C) = H(K) + H(M)$$

hence

$$n4.7 = H(K) + nF$$

If we solve for n we conclude

$$n = \frac{H(K)}{4.7 - F}$$

For a known plaintext attack we have the following theorem.

Theorem

$$H(K|C, M) = H(K) - H(C|M)$$

**Proof**: Applying the identity H(X, Y) = H(X) + H(Y|X) we have H(K, C, M) = H(C, M) + H(K|C, M) = H(M) + H(C|M) + H(K|C, M)

on the other hand

$$H(K, C, M) = H(K, M) = H(K) + H(M)$$

since K and M are independent. Hence

H(M) + H(C|M) + H(K|C,M) = H(K) + H(M)

and solving for H(K|C, M) yields the identity.