- Entropy of random letters with probability of each letter chosen $1 / 26$

$$
\sum_{\alpha=A}^{Z} 1 / 26 \log _{2}(26) \approx 4.7
$$

- Entropy of letters in English chosen independently using single letter probabilities $p_{\alpha}$

$$
\sum_{\alpha=A}^{z} p_{\alpha} \log _{2}\left(1 / p_{\alpha}\right) \approx 4.16
$$

- Entropy of English using biletter statistics $\approx 3.2$
- Experimental entropy of English with at least 25 letters $\approx 1.2$

Let $F$ represent the of number of bits of information per English letter of text.

|  | $n<8$ | $F=4.16$ |
| :---: | :---: | :--- |
| General rules | $8<n \leq 15$ | $F=3.2$ |
|  | $15<n \leq 25$ | $F=2$ |
|  | $25<n$ | $F=1.2$ |

For a cyphertext only attack to estimate the unicity distance (set $H(K \mid C)=0$ ) we use the equation

$$
H(C)=H(K)+H(M)
$$

hence

$$
n 4.7=H(K)+n F
$$

If we solve for $n$ we conclude

$$
n=\frac{H(K)}{4.7-F}
$$

For a known plaintext attack we have the following theorem.

## Theorem

$$
H(K \mid C, M)=H(K)-H(C \mid M)
$$

Proof: Applying the identity $H(X, Y)=H(X)+H(Y \mid X)$ we have $H(K, C, M)=H(C, M)+H(K \mid C, M)=H(M)+H(C \mid M)+H(K \mid C, M)$ on the other hand

$$
H(K, C, M)=H(K, M)=H(K)+H(M)
$$

since $K$ and $M$ are independent. Hence

$$
H(M)+H(C \mid M)+H(K \mid C, M)=H(K)+H(M)
$$

and solving for $H(K \mid C, M)$ yields the identity.

