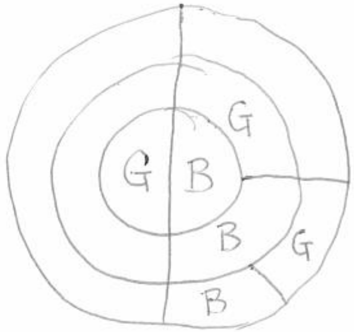
A country wishes to increase the percent~~a~~age of children that are born male so they adopt the following law.

"A married couple is to have children until either they have one girl or three boys."

What this means is that each couple is required to have at least one child. If the first child is a girl, then the couple will not have any more children. If the first child is a boy, the couple has a second child. If the second child is a boy they have a third, otherwise the couple stops having children.

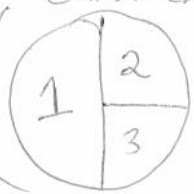
What is the probability that a couple has more boys than girls? What is the probability that a couple has more girls than boys? How many children are expected on average? How many boys are expected on average? How many girls are expected on average?

more girls than boys vs. more boys than girls

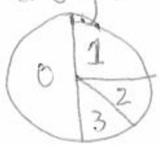


P(more girls than boys) = 1/2 P(more boys than girls) = 1/4

children

E() $= 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} + 3\frac{1}{4} = \frac{7}{4}$

boys

E() $= 0\cdot\frac{1}{2} + 1\cdot\frac{1}{4} + 2\cdot\frac{1}{8} + 3\cdot\frac{1}{8} = 7/8$

girls

E() $= 1\cdot\frac{7}{8} + 0\cdot 1/8 = 7/8$

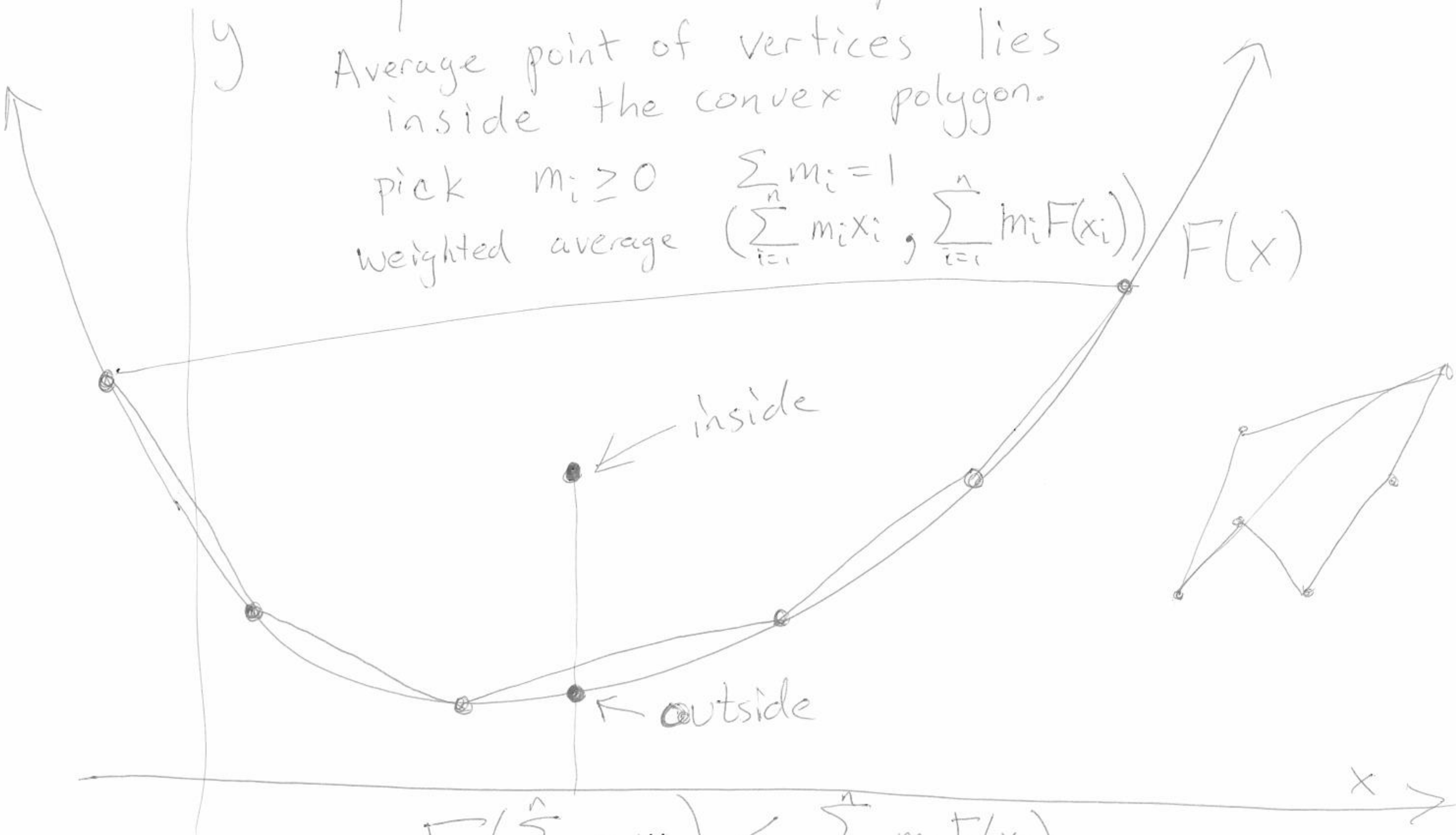
$F(x)$ is concave up

pick a bunch of points x_i

Average point of vertices lies inside the convex polygon.

pick $m_i \geq 0$ $\quad \sum_n m_i = 1$

weighted average $\left(\sum_{i=1}^{n} m_i x_i , \sum_{i=1}^{n} m_i F(x_i)\right)$



$$F\left(\sum_{i=1}^{n} m_i x_i\right) \leq \sum_{i=1}^{n} m_i F(x_i)$$

Example: $y = x^2$

choose a bunch of $x_1, x_2, \ldots, x_n$

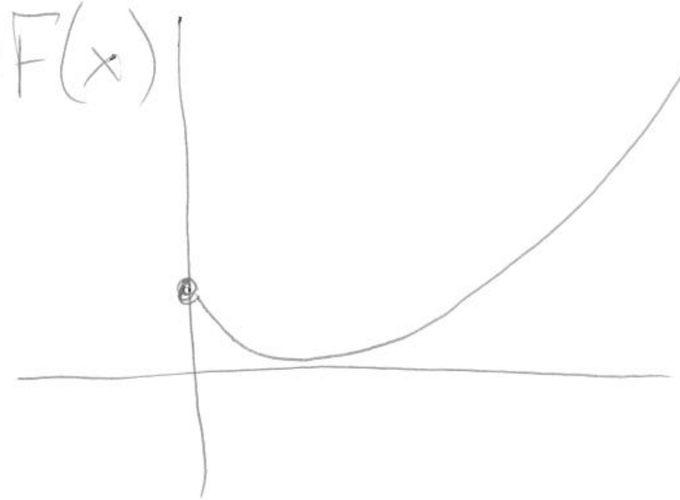
and $m_i = 1/n$

$$\left(x_1/n + x_2/n + \cdots + x_n/n\right)^2 \le x_1^2/n + x_2^2/n + \cdots + x_n^2/n$$

$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \le \sqrt{\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}}$$

arithmetic mean $\le$ geometric mean

$$y = x \log(x) = F(x)$$



Choose $x_i = p_i/q_i$ where p_i & q_i are probabilities

$m_i = q_i$ $\qquad \sum p_i = 1$

$\sum q_i = 1$

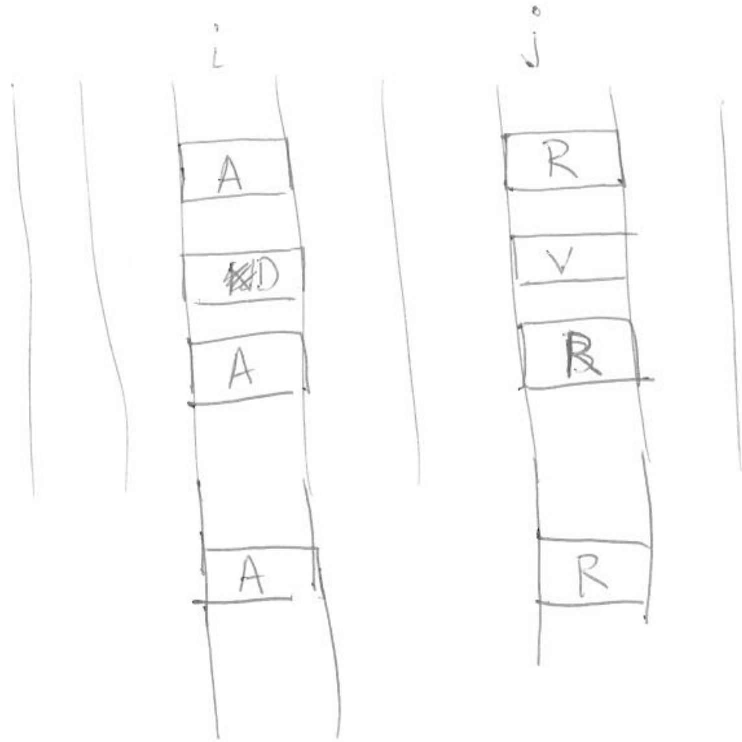
$$F\left(\sum_{i=1}^{n} x_i m_i\right) = F\left(\sum_{i=1}^{n} (p_i/q_i) q_i\right) = F\left(\sum_{i=1}^{n} p_i\right) = F(1) = 0$$

$$\sum_{i=1}^{n} q_i F(p_i/q_i) = \sum_{i=1}^{n} \cancel{q_i} (p_i/\cancel{q_i}) \log(p_i/q_i)$$

Identity shows

$$0 \le \sum_{i=1}^{n} p_i \log(p_i/q_i) = \sum_{i=1}^{n} p_i(\log p_i - \log q_i)$$

$$\sum_{i=1}^{n} p_i \log q_i \le \sum_{i=1}^{n} p_i \log p_i$$



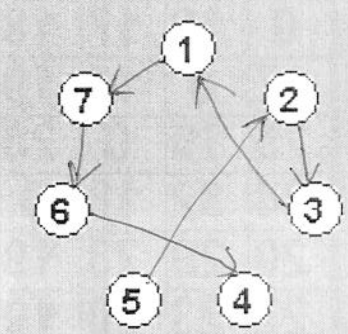
$$N_{AR}^{(i,j)} = 3$$

$$N_{DV}^{(i,j)} = 1$$

Example of table of $\sum_{a,b=A}^{Z} p_{a,b} \log N_{a,b}^{(i,j)}$ with correct period

We should see high values in each row and column except one row (the last position of the permutation) and one column (the first position of the permutation).

0	26	31	34	26	20	36
18	0	53	32	24	32	27
39	26	0	26	24	29	18
27	19	33	0	26	28	22
24	39	29	29	0	26	21
21	28	28	44	27	0	23
29	26	28	23	25	43	0



PERMUTATION

Compute for each i & j

$\cancel{\sum_{a,b} N_{ab}^{(i,j)}}$

Read column 5 then 2 then 3 then 1 then 7 then 6 then 4

5231764 is the decrypting permutation.