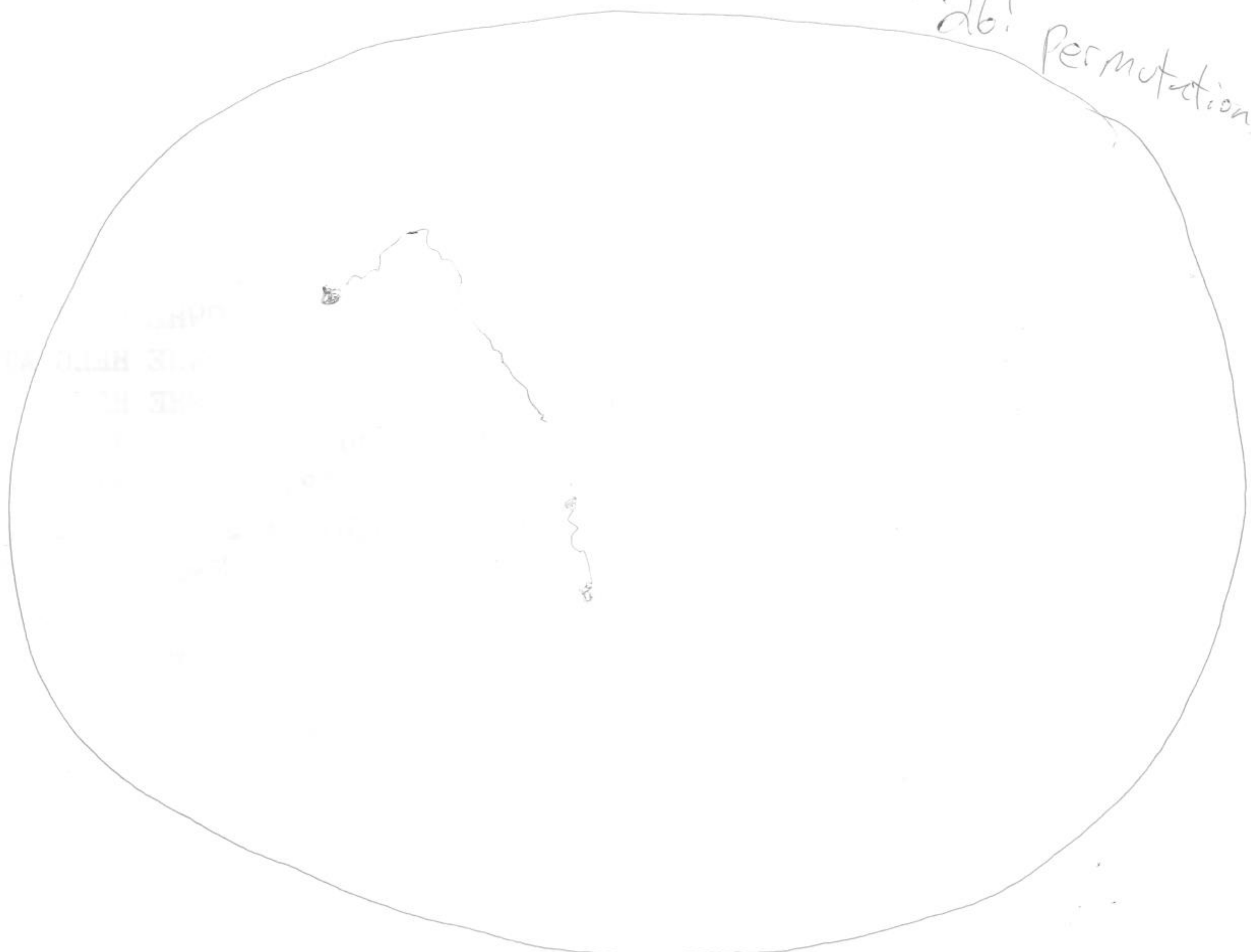
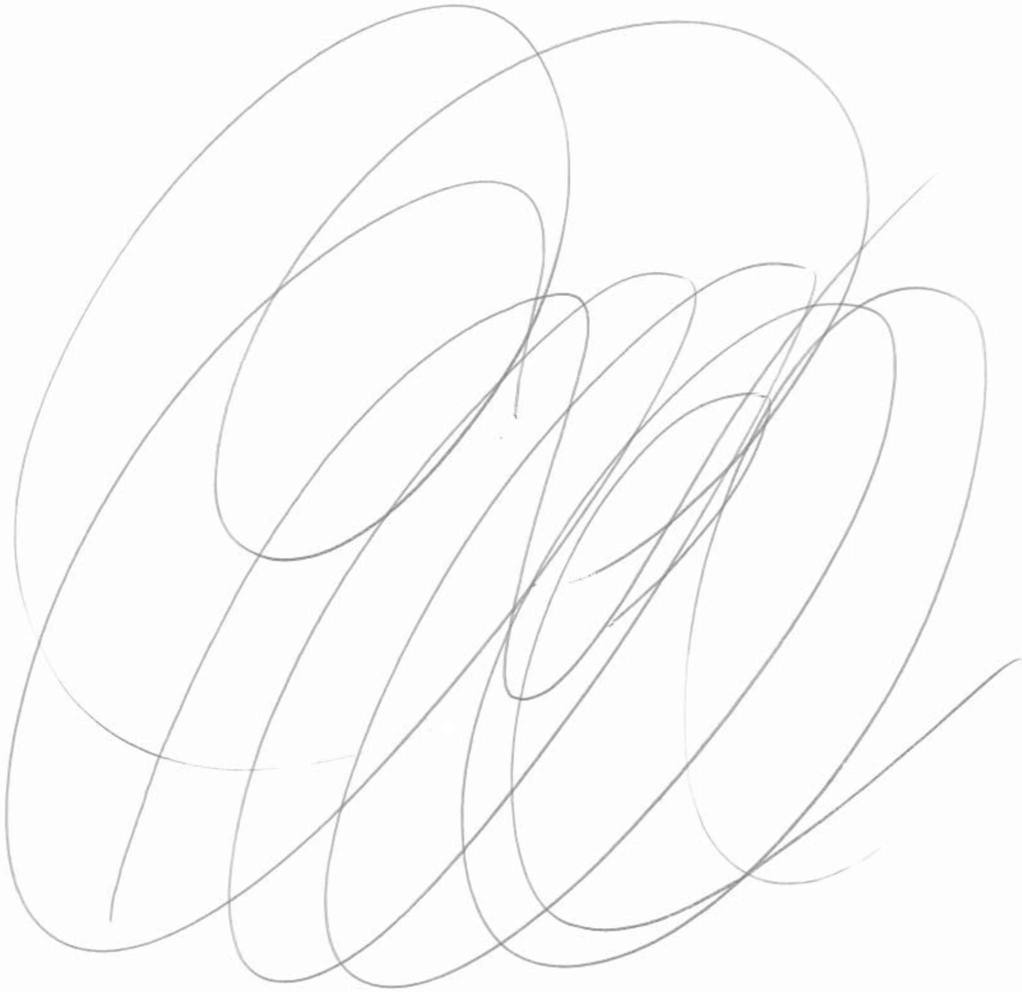


26! permutations

PMZMRQ

Q	N	A	N	S
R	O	B	O	T
S	P	C	P	V
T	Q	D	Q	
U	R	E	R	
V	S	F	S	
W	T	G	T	
X	U	H	U	
Y	V	I	V	
Z	W	J	W	
A	X	K	X	
B	Y	L	Y	
C	Z	M	Z	
D	A	N	A	
E	B	O	B	
F	C	P	C	
G	D	Q	D	
H	E	R	E	
I	F	S	F	
J	G	T	G	
K	H	U	H	
L	I	V	I	
M	J	W	J	
N	K	X	K	
O	L	Y	L	



The Entropy of An Event

Definition: The *entropy of an event A* is:

1. the measure of uncertainty we *feel* about the occurrence of A.
2. the amount of *information*, measured in bits, contained by A.

$$\boxed{\text{Events that occur with equal probability have the same amount of uncertainty and contain the same amount of information}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l}\text{The entropy of an event should be a function of the probability of that event occurring} \\ \\ \qquad \text{The entropy of event } A = h(P(A))\end{array}}$$

Information is a function of the probability.

$$\boxed{\text{What properties should the entropy function, } h\text{, have to numerically express the measure of our uncertainty about the occurrence of an event in a manner which is compatible with our intuitive notion of uncertainty?}}$$

Information Theory Definitions

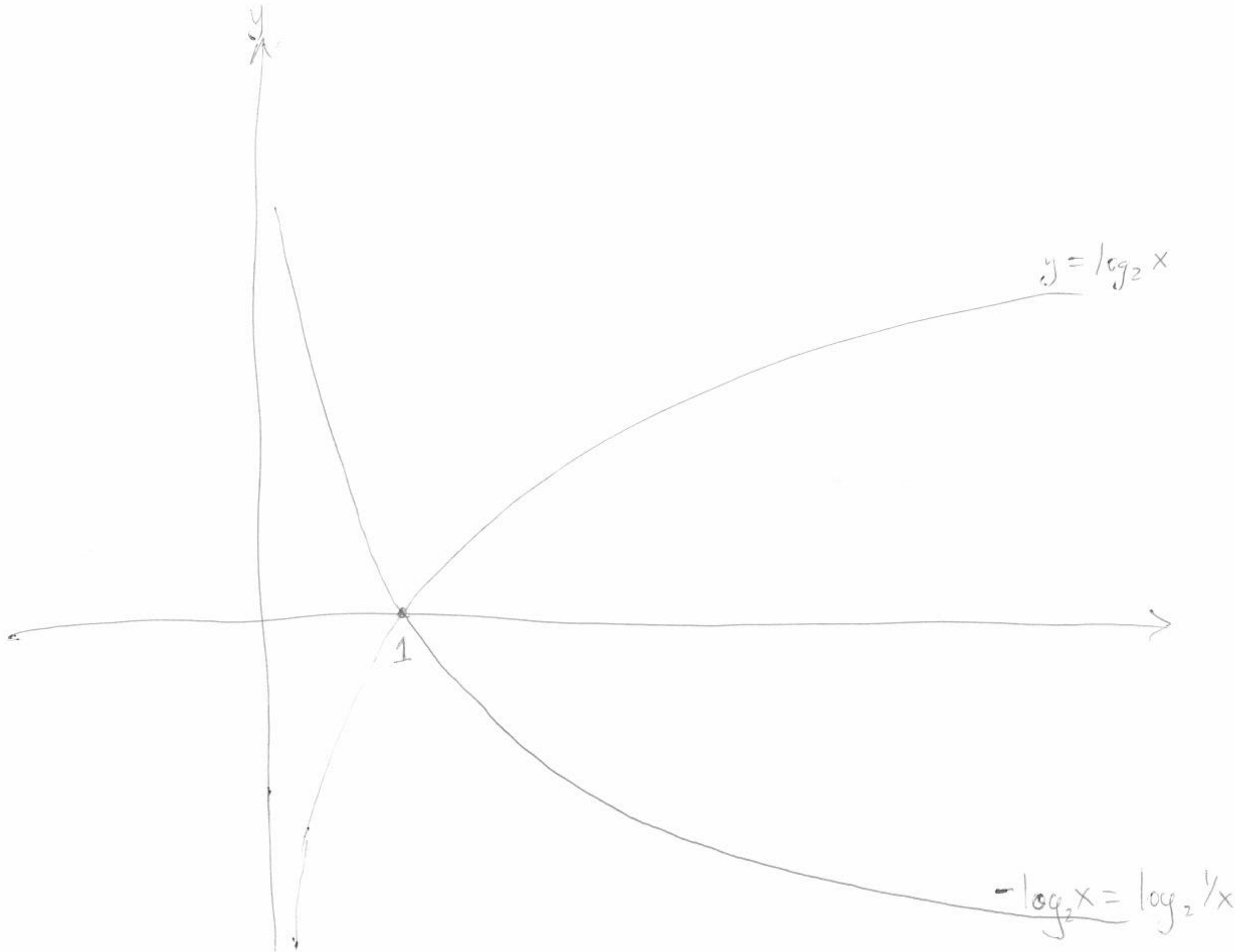
Definition: The Entropy of a random variable X

$$H(X) = \sum_{a} P[X = a] \ log_2 \left(\frac{1}{P[X = a]} \right)$$

Expected value of the entropy function of X

Definition: The entropy of two random variables X and Y.

$$H(X,Y) = \sum_{a,b} P[X = a \ \& \ Y = b] \ log_2 \left(\frac{1}{P[X = a \ \& \ Y = b]} \right)$$


y
y = log₂ x
1
-log₂ x = log₂ 1/x

If X & Y are independent, then

$$P(X=a \,\&\, Y=b) = P(X=a)\,P(Y=b)$$

$$\begin{aligned}
H(X,Y) &= \sum_{a,b} P(X=a \,\&\, Y=b) \log_2\left(\frac{1}{P(X=a \,\&\, Y=b)}\right) \\
&= \sum_{a,b} P(X=a)P(Y=b) \log_2\left(\frac{1}{P(X=a)P(Y=b)}\right) \\
&= \sum_{a,b} P(X=a)P(Y=b)\left(\log_2\left(\frac{1}{P(X=a)}\right) + \log_2\left(\frac{1}{P(Y=b)}\right)\right) \\
&= \sum_{a,b} P(X=a)P(Y=b)\log_2\left(\frac{1}{P(X=a)}\right) + \sum_{a,b} P(X=a)P(Y=b)\log_2\left(\frac{1}{P(Y=b)}\right) \\
&= \sum_{a} P(X=a)\log_2\left(\frac{1}{P(X=a)}\right) + \sum_{b} P(Y=b)\log_2\left(\frac{1}{P(Y=b)}\right) \\
&= H(X) + H(Y)
\end{aligned}$$

Intuition says if X & Y are not indep

then $H(X,Y) \leq H(X) + H(Y)$