

9
coins - visually identical

one weighs more than the other \leq

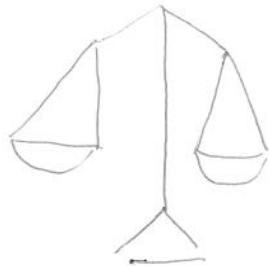
Devise a scheme for weighing so that the maximum number of weighings on a balance scale is as small as possible

each weighing

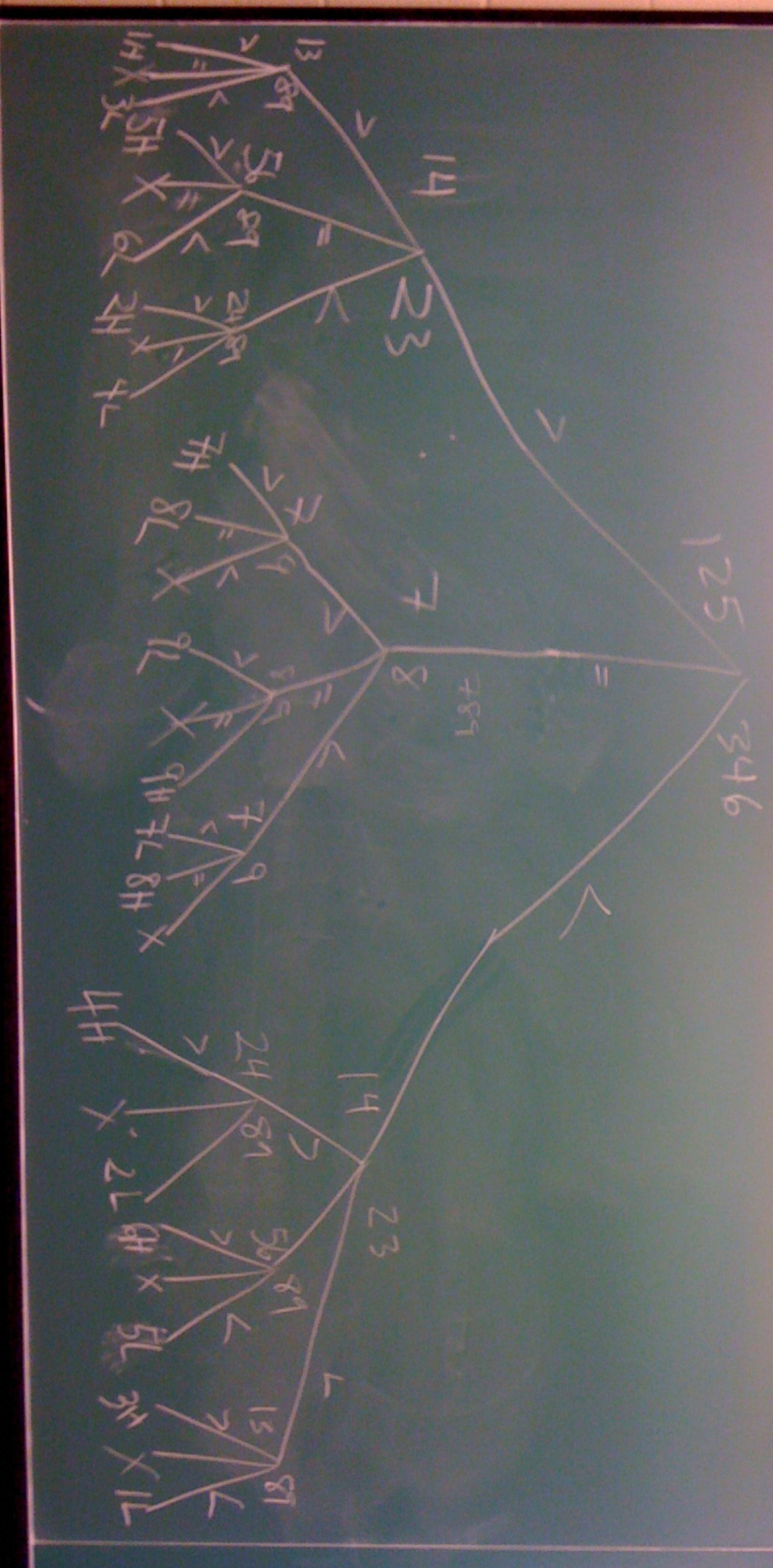


≈ 1.7

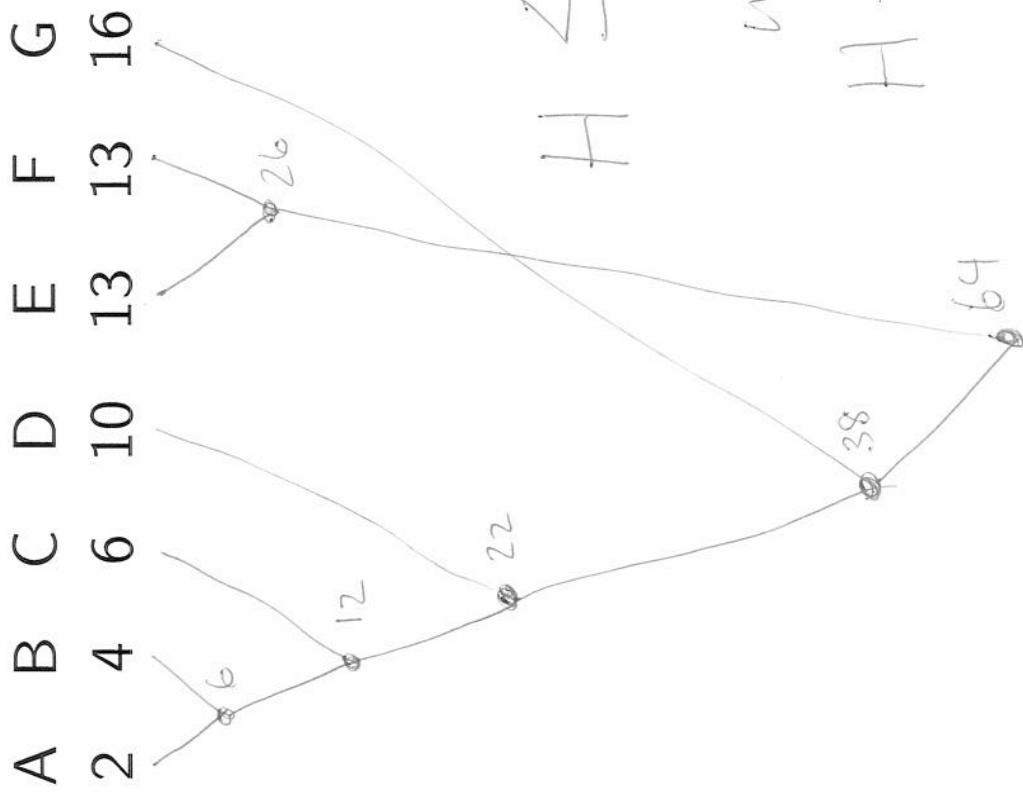
Learn $\leq \log_2 3$ bits of information



The location of coin + if it is more or less has 14 possible outcomes. Outcome stored in $\log_2 14$ bits of information
 ≈ 3.8



Huffman Code



$$ECL_{\text{of}} \leq H + 1$$

where

H = entropy

Want a code s.t. expected code length is as small as possible.

Huffman Code

Begin with a text file with the following frequencies

letter	A	B	C	D	E	F	G
frequency	2	4	6	10	13	13	16

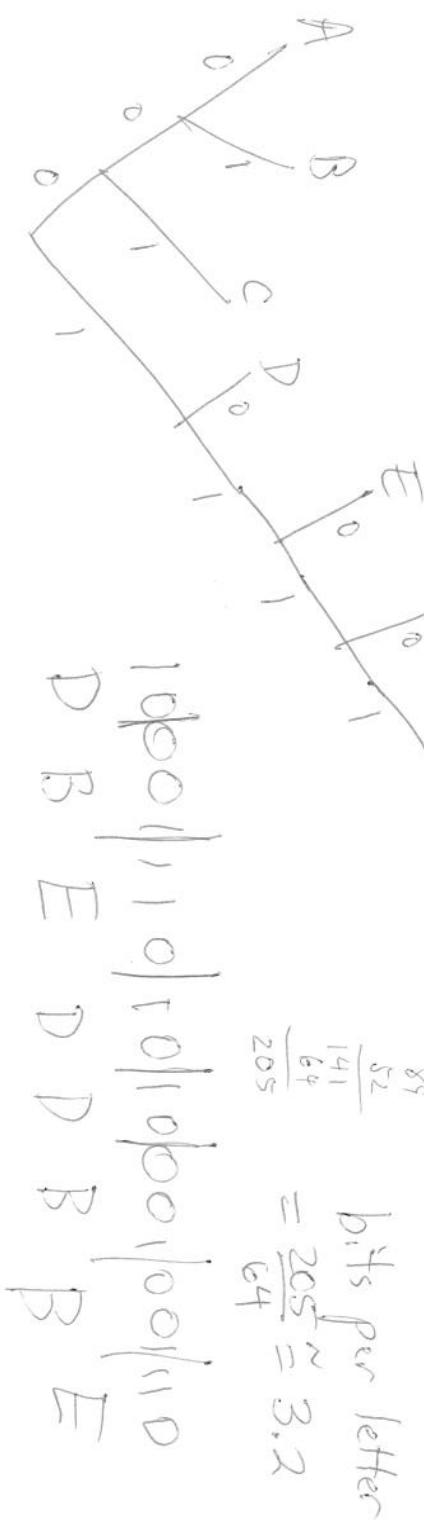
Note # of bits

$$= 2 \cdot 3 + 4 \cdot 3 + 6 \cdot 2 + 2 \cdot 10$$

$$+ 3 \cdot 13 + 4 \cdot 13 + 4 \cdot 16 = 205$$

bits per letter

$$\frac{141}{64} = \frac{205}{64} \approx 3.2$$



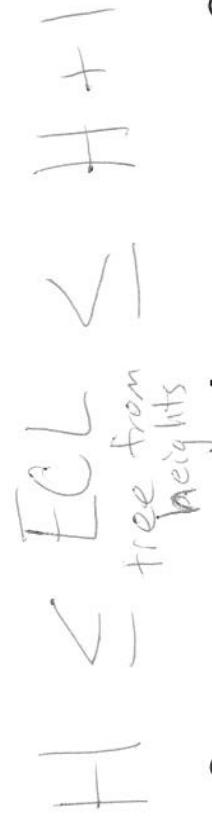
Tree from heights

Begin with a text file with the following frequencies

letter	A	B	C	D	E	F	G
frequency	2	4	6	10	13	13	16
code length	4	4	3	3	3	2	2

$$\text{average bits per letter} = (4 \cdot 2 + 4 \cdot 4 + 3 \cdot 6 + 3 \cdot 10$$

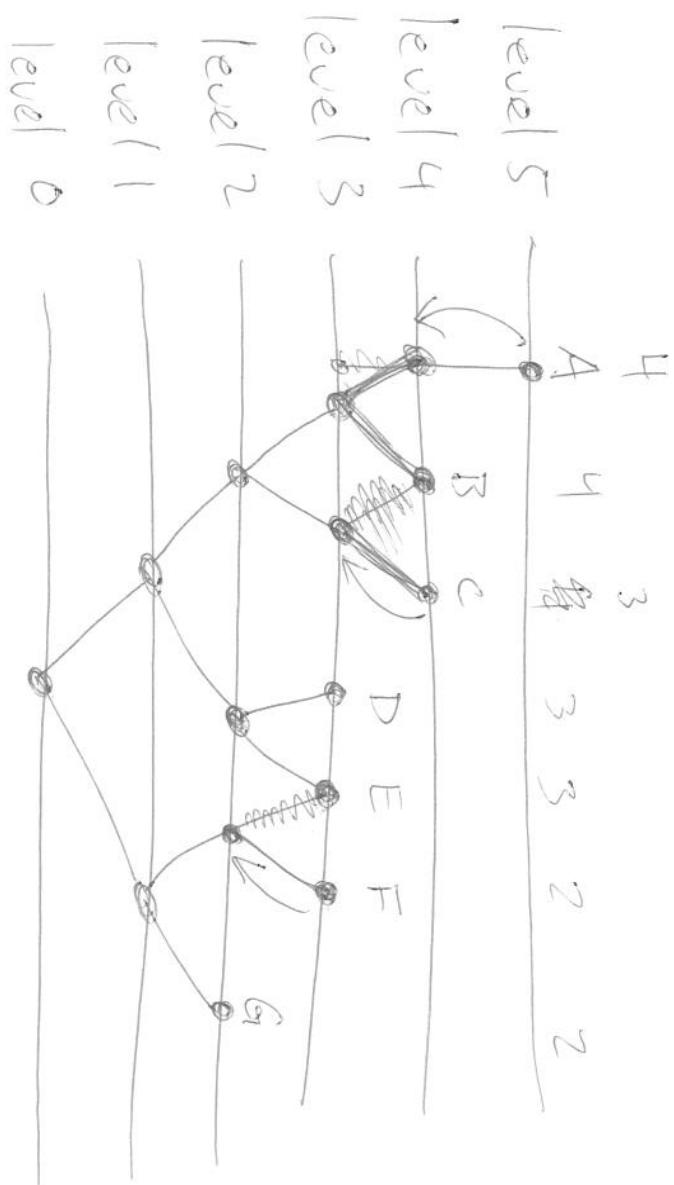
$$+ 3 \cdot 13 + 2 \cdot 13 + 2 \cdot 16) / 64 = \frac{169}{64} \approx 2.641$$



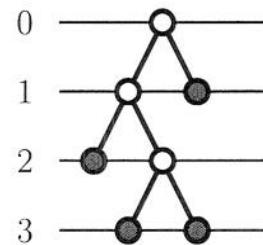
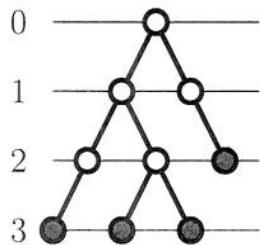
$$\begin{aligned}\text{Entropy} &= \frac{2}{64} \log_2(32) + \frac{4}{64} \log_2(16) + \frac{6}{64} \log_2\left(\frac{64}{6}\right) \\ &\quad + \frac{10}{64} \log_2\left(\frac{64}{10}\right) + 2 \times \frac{13}{64} \log_2\left(\frac{64}{13}\right) + \frac{1}{4} \log_2(4) \approx 2.579\end{aligned}$$

Tree from Heights

α	A	B	C	D	E	F	G
ρ_α	$\frac{2}{64}$	$\frac{4}{64}$	$\frac{6}{64}$	$\frac{10}{64}$	$\frac{13}{64}$	$\frac{13}{64}$	$\frac{16}{64}$
$\lceil \log_2(\frac{1}{\rho_\alpha}) \rceil$	5	4	4	3	3	3	2



Leaf Heights



$$\frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^2} = 5/8$$

$$\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^1} = 1$$

Theorem 1 The sequence of integers h_1, h_2, \dots, h_n are leaf heights of a binary tree if and only if

$$\sum_{i=1}^n \frac{1}{2^{h_i}} \leq 1$$

with equality only if the tree is complete.

Proof:

$$\sum_{i=1}^n \frac{1}{2^{h_i}} = 1$$

level $\in k$

level 0

$$\sum_{i=1}^n \frac{1}{2^{h_i}} = \frac{1}{2^k} < 1$$

$$\sum \frac{1}{2^{h_i}} \leq 1$$

