

Homophonic Substitution

S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1	2	3	4	5	6	7	8		
T	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1	2	3	4	5	6	7	
A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
N	14	15	16	17	18	19	20	21	22	23	24	25	1	2	3	4	5	6	7	8	9	10	11	12	13	

Start with 1

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1	2	3	4	5	6	7	8	
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	26	27	28	29	30	31	32	
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	
89	90	91	92	93	94	95	96	97	98	99	100	76	77	78	79	80	81	82	83	84	85	86	87	88	

← 1-25
 ← 26-50
 ← 51-75
 ← 76-100

Key: STAN

Plaintext #1: T H E N W H Y D I D Y O U T
 82-16-55-45
 U R N S O M E O F U S I N S

I D E O U T ?

Cyphertext #2: 69-16-9-2-85-33-81-35-51-25-61-40-13-1-45-93-85-20-64-77
 T H A T W A S C A R L H E S N E W M O O

ADFGVX

The ADFGVX system was first used in the battlefield march 5th 1918. Was broken June 1st by Georges Painvin

K1: A one square
 K2: a permutation of n (n even)

A	D	F	G	V	X
C	O	8	X	F	4
M	K	3	A	Z	9
N	W	L	0	J	D
S	I	Y	H	U	
P	1	V	B	6	R
E	Q	7	T	2	G

← first key
 ← key

4	9	5	15	2	8	16	12	13	17	1	18	3	19	10	7	6	11	14	20
G	I	V	X	D	P	V	X	X	A	X	D	G	I	X	A				
H	Q	R	E	E	Q	U													
A	V	V	X	A	D	E	A	X	G	E	E	G	I	F					
F	R	O	N	T	L	I	N	E	S										
G	I	F	X	G	G	X	D	G	X	G	G	I	F	A	D				
I	T	U	A	A	T	I	O	N	B	Y									
X	I	G	X	A	F	E	X	A	X	A	V	X	D	G					
T	E	L	E	G	R	A	M	H	Q										
X	I	F	X	G	G	V	A	A	A	D	V	X	V	A					
7	T	H	C	O	R	P	S	E	E	D									

← second key

← encrypted message
 ← message

Hill Encipherment

Key: a kxk matrix

ALL ARITHMETIC IS DONE (MOD 26)

$$A = \begin{bmatrix} 11 & 2 \\ 1 & 5 \end{bmatrix} \quad k=2 \quad A^{-1} = \begin{bmatrix} 5 & -2 \\ -1 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 24 \\ 25 & 11 \end{bmatrix}$$

$$\det A = 11 \cdot 5 - 2 \cdot 1 = 53 \equiv 1 \pmod{26}$$

$$I \cdot ((11 \cdot 5 - 2 \cdot 1) \equiv 1 \pmod{26})$$

$$\begin{bmatrix} 11 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 5 & 24 \\ 25 & 11 \end{bmatrix} = \begin{bmatrix} 55+50 & 264+22 \\ 5+125 & 24+55 \end{bmatrix}$$

A^{-1}

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Playfair Cipher

Key

D	E	N	I	A
L	B	C	F	G
H	K	M	Q	P
Q	R	S	T	U
V	W	X	Y	Z

Plaintext:

382 Robertson Dr, West Hollywood

TH RE EQ EI GH TQ TW OR OB ER TS ON
 QO WB DR NA LP UR RY KT KF BW UT MT
 DR WE ST HO LQ LY WO OD
 EB TV KP HV FV YK HT
 EQ

Plaintext: MEAT

Numerical: 12-4 0-19

$$A^* \text{plaintext: } \begin{bmatrix} 11 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 11 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 19 \end{bmatrix}$$

Cyphertext: 11-12+2-4 12-1+4-5 2-11 5-19

Cyphertext: WU UO EI AY

Numerical: 22-20 20-14 4-8 0-24

$$A^* \text{Cyphertext: } \begin{bmatrix} 5 & -2 \\ -1 & 11 \end{bmatrix} \begin{bmatrix} 22 & 20 \\ 20 & 14 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 0 & 24 \end{bmatrix}$$

Plaintext: 5-22-2-20 -1-22+11-20 5-20-2-14

$$A^{-1} \cdot A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-1 \cdot 20 + 11 \cdot 14$$

$$5 \cdot 4 - 2 \cdot 8$$

$$-1 \cdot 4 + 11 \cdot 8$$

$$-2 \cdot 24$$

$$11 \cdot 24$$

$$\frac{198}{182}$$

$$\frac{7}{16}$$

95-78
17

$A^{-1} \cdot \text{cyphertext} \rightarrow \text{KGMR}$

$$\equiv A^{-1} \cdot A \cdot \text{plaintext}$$

$$\equiv I \cdot \text{plaintext}$$

$$\equiv \text{plaintext}$$

220

84

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \cdot \frac{1}{ad-bc}$$

$$\begin{aligned} A^{-1} \cdot A &= \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} \frac{ad-bc}{ad-bc} & \frac{bd-bd}{ad-bc} \\ \frac{-ac+ac}{ad-bc} & \frac{-bc+ad}{ad-bc} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Mod 26

Find e s.t.

$$(ad-bc) \cdot e \equiv 1 \pmod{26}$$

$$e \equiv (ad-bc)^{-1} \pmod{26}$$

And

$$A^{-1} \equiv \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \cdot e \pmod{26}$$

then

$$A^{-1} \cdot A \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \pmod{26}$$

Sometimes

$\det A$ will have a common factor with the modulus 26.

if it does then there will not be a value e st.

$$e \cdot \det A \equiv 1 \pmod{26}$$

the inverse of A does not exist
By working ^{in this case} mod 29,

then $\det A$ only has a common factor with 29 if it is a multiple of 29.

A → 0

B → 1

C → 2

⋮

Z → 25

Ø → 26

1 → 27

2 → 28

If A doesn't have an inverse the plaintext may not be recoverable from the cyphertext.

← choose a convention that the last 3 symbols go to 26, 27, 28