CONDITIONAL PROBABILITY: If A and B are events the ratio

The probability of an event A happens given that an event B did happen. $\longrightarrow$

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

$P(\text{roll}=2 \mid \text{roll even}) = \frac{P(\text{roll}=2 \,\&\, \text{even})}{P(\text{roll even})} = \frac{1/6}{1/2} = \ldots$

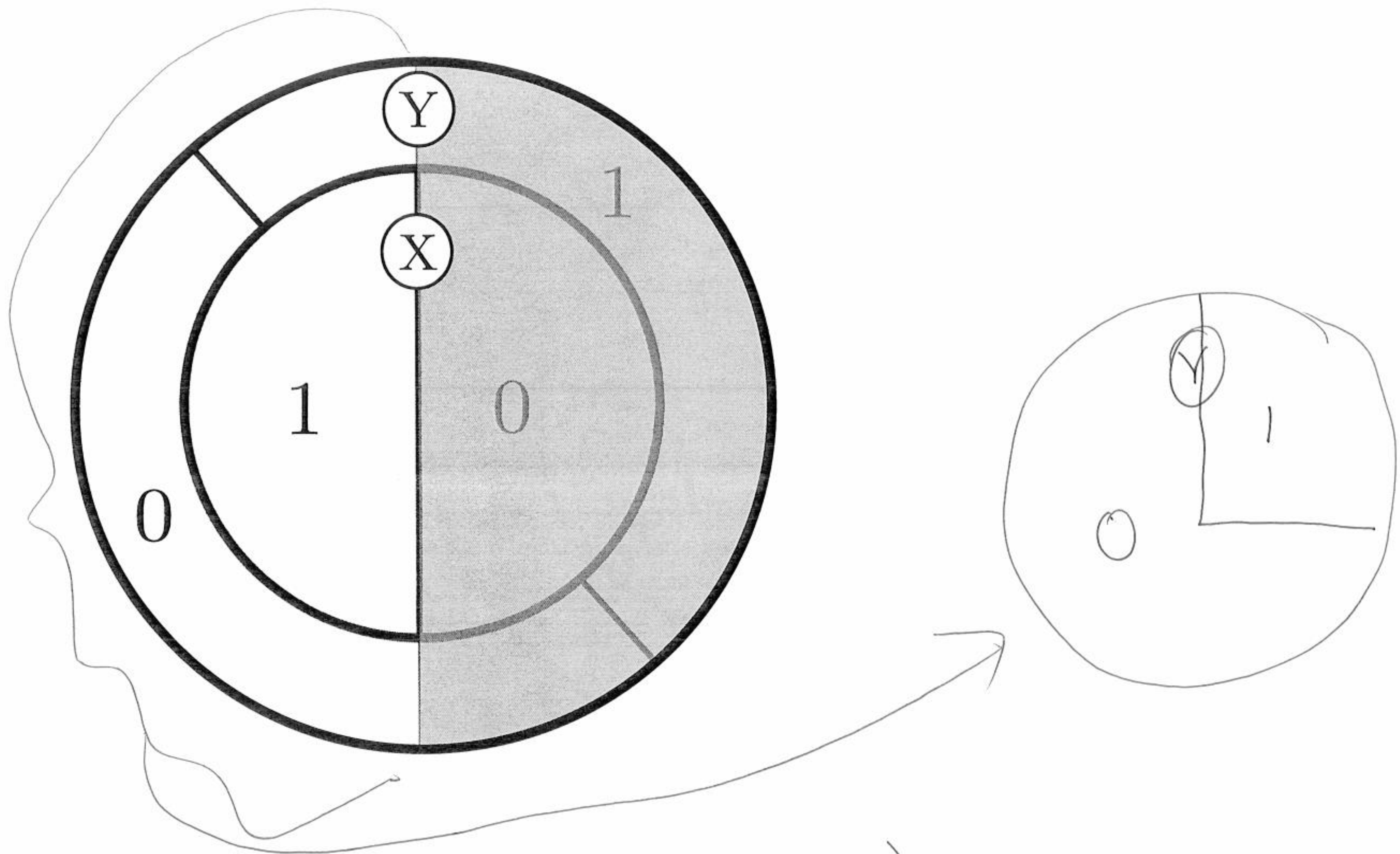
$P(\text{roll}=2 \mid \text{roll odd}) = \frac{P(\text{roll}=2 \,\&\, \text{odd})}{P(\text{odd})}$

is usually referred to as the CONDITIONAL PROBABILITY OF A GIVEN B. The concept arises as follows. Given the event B we can construct a new experiment E_B by carrying out E and recording its outcome only when it falls in B. We can argue that the probability of A under E_B will be the expression above where $P[A \cap B]$ and P[B] are the probabilities of $A \cap B$ and B under E. We shall refer to E_B as E CRIPPLED by B.

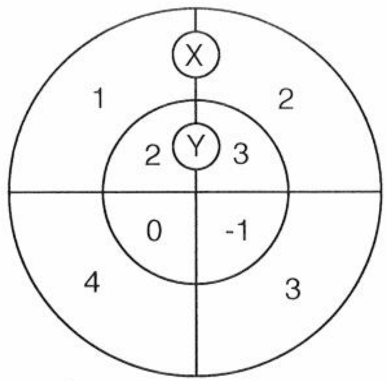
CONDITIONAL EXPECTATION OF A RANDOM VARIABLE: Given an event B, if we carry out the crippled experiment E_B instead of E , then all the probabilities change and so do all expectations. If X is a random variable and the events $A_1, A_2, \ldots, A_k$ decompose Ω as before then expression

$$E[X|B] = x_1 P[A_1|B] + x_2 P[A_2|B] + \cdots + x_k P[A_k|B]$$

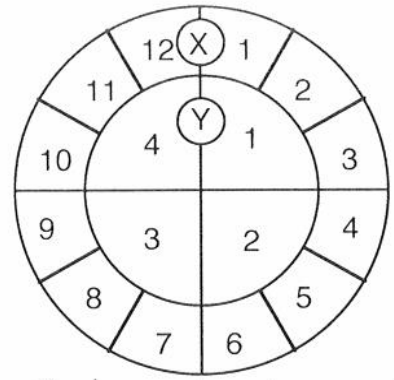
gives the expected value of X under E_B. We refer to it as the CONDITIONAL EXPECTATION OF X GIVEN B.



$$P(Y=0 \mid X=1) = \frac{P(Y=0 \,\&\, X=1)}{P(X=1)} = \frac{3/8}{1/2} = \frac{3}{4}$$

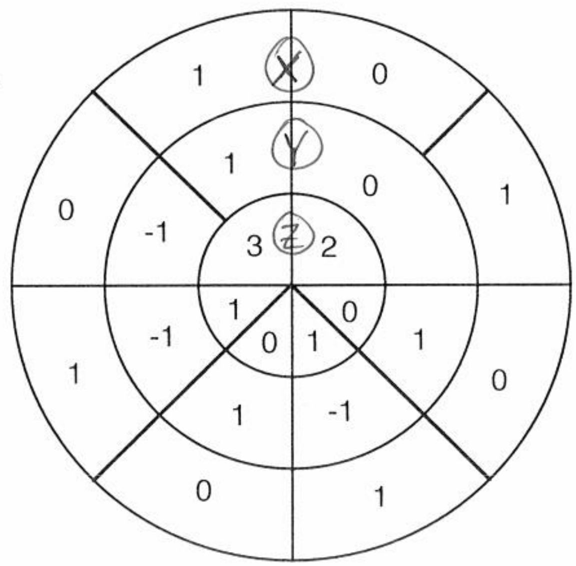


X is dependent on X
X is dependent on Y



X is not dependent on Y
Y is dependent on X

Z is dependent on X & Y
Y is dependent on X & Z
X is not dependent on Y & Z



Y is not dependent on X
Z is not dependent on X
X is not dependent on Y
Z is not dependent on Y
X is not dependent on Z
Y is not dependent on Z

X is dependent on Y if X is a function of Y

that is, knowing the value of Y determines the value of X

DEPENDENCE: The random variable Y is said to be DEPENDENT upon the random variable X if and only if Y is a function of X . Similarly we say that Y is dependent upon X_1 , X_2 , . . . , X_n if for some function $f(x_1, x_2, \ldots, x_n)$ we have

$$Y = f(X_1, X_2, \ldots, X_n)$$

INDEPENDENCE: In probability theory, “independence” is not the negation of “dependence” We say that X is “independent” of Y only if knowing the value of Y “doesn’t change our uncertainty” about X. More precisely, if we cripple our experiment E by any of the events [Y = b] the probabilities of all the events [X = a] do not change. Mathematically this is translated in the conditions that for all choices of a and b

$$P(X = a|Y = b) = P(X = a)$$

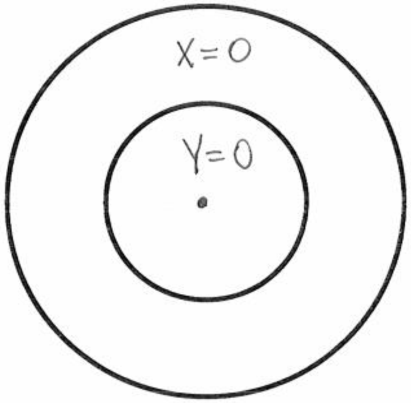
this simply means that $\frac{P(X=a \,\&\, Y=b)}{P(Y=b)} = P(X=a)$

$$P(X = a \text{ and } Y = b) = P(X = a)P(Y = b)$$

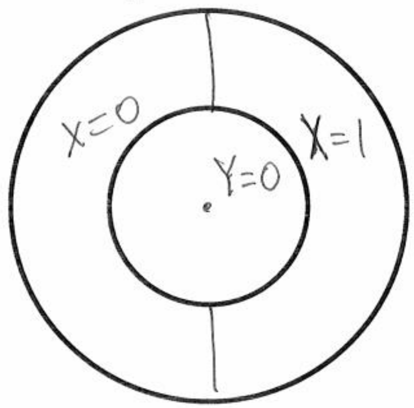
"X is dependent on Y" and "X is independent of Y" are not opposite statements of each other, rather they are on opposite sides of a spectrum of possibilities.

"X is not dependent on Y" does not mean "X is independent of Y"

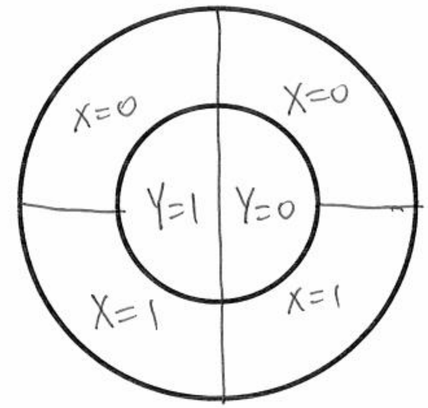
X is independent of Y
X is dependent on Y
Y is dependent on X



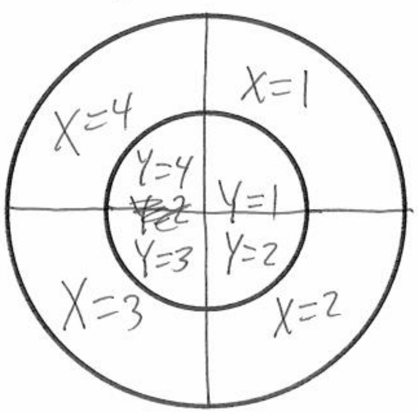
X is independent of Y
X is not dependent on Y ✓
Y is dependent on X ✓



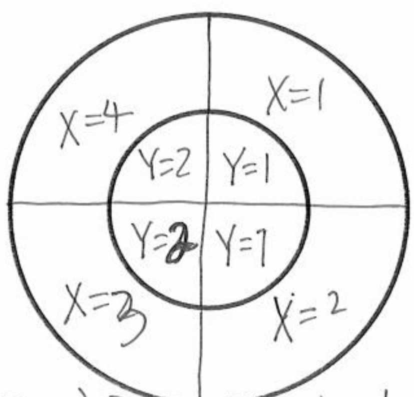
X is independent of Y
X is not dependent on Y
Y is not dependent on X



X is not independent of Y
X is dependent on Y
Y is dependent on X



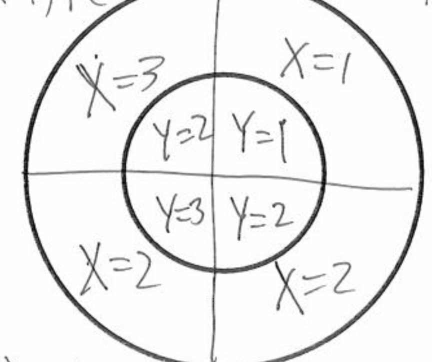
X is not independent of Y ✓
X is not dependent on Y ✓
Y is dependent on X ✓



$P(X=1)P(Y=1) = \frac{1}{4}\cdot\frac{1}{2} \neq P(X=1 \& Y=1) = 1/4$

X is not independent of Y
X is not dependent on Y
Y is not dependent on X

$P(X=1)P(Y=\,) = 1/4 \cdot 1/4 \neq P(X=1 \& Y=$
$= 1/4$



A) 95%

B) 90%

C) 75%

D) 60%

E) 50%

$$P(\text{patient has disease} \mid \text{patient tests positive}) =$$

$$\frac{P(\text{has \& tests positive})}{P(\text{tests positive})} = \frac{(.05)\times(.95)}{(.95)\times(.05)+(.05)\times(.95)}$$

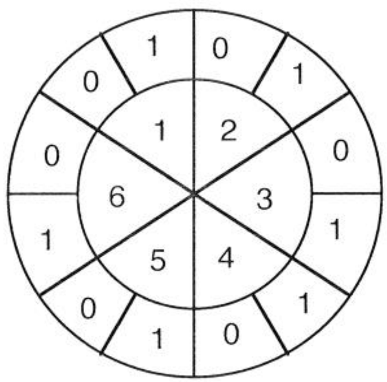
$$= 1/2$$

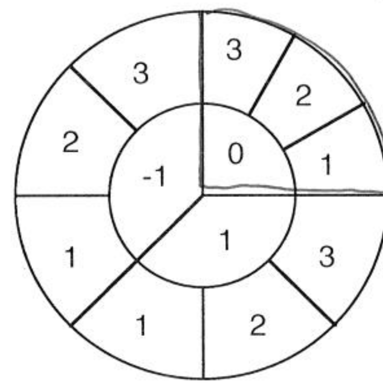
X is independent of Y if $P(X = a | Y = b) = P(X = a)$

or $P(X = a \text{ and } Y = b) = P(X = a)P(Y = b)$

or knowing the value of Y does not change the probabilities of X

If X is independent of Y, then Y is independent of X.





$P(Y=1) = 1/3$

$P(Y=1 | X=0) = 1/3$

$= \frac{P(Y=1 \& X=}{P(X=}$

$= \frac{1/12}{1/4}$