(1) Recall the identity that we use to prove the logarithm inequality
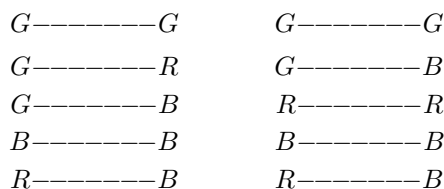
$$\sum_i m_i f(x_i) \le \ \text{or} \ \ge f\left(\sum_i m_i x_i\right)$$

(the direction of $\le, \ge$ depends if f is concave up or down). Use this formula to derive an inequality of the form

$$\frac{1}{n}(\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}) \le c\sqrt{n+1}$$

for some appropriate value of c which does not depend on n. That is, choose some values $x_1, x_2, \ldots, x_k$ and weights $m_1, m_2, \ldots, m_k \ge 0$ where $\sum_i m_i = 1$ and an appropriate function $f(x)$ (state clearly if your function is concave up or down) to see how the left and right hand side of the expressions above compare. You may need the identity that $\sum_{i=1}^{n} i = n(n+1)/2$.

(2) 10 sticks are placed in a bag and each end is colored with the colors Red, Green or Blue with the colors indicated in the diagram below. If I reach in the bag and pull out a stick with one end in my fist and the other end showing, what is the probability that the end that is in my fist is green given that the end showing is green?

$$
\begin{array}{ll}
G\text{--------}G \qquad & G\text{--------}G \\
G\text{--------}R \qquad & G\text{--------}B \\
G\text{--------}B \qquad & R\text{--------}R \\
B\text{--------}B \qquad & B\text{--------}B \\
R\text{--------}B \qquad & R\text{--------}B
\end{array}
$$

(3) The following cyphertext was encrypted with the Vigenere system with key length 8. Below is a histogram of the number of letters that appear in the cyphertext superimposed with the histogram of the distribution of English letter statistics in each of the positions modulo 8. Find the first full word of plaintext and the key.

XEKBP GWAKR JYGVG TDKAC KXKQD LITCM FKIVD VNVGT

DKAJJ SSJTK ZLKRY LDBQY NCGML YQPJM KOWPB UGCSJ