The Jargon of Probability

EXPERIMENT, RANDOM VARIABLES: This refers to an activity, not necessarily scientific, which involves the production of data some of which are "random". We denote an experiment by $\mathcal{E}$ and the data by $X, Y, Z, \ldots$. The latter are usually referred to as the RANDOM VARIABLES associated with $\mathcal{E}$.

RANDOM, SAMPLE SPACE, PROBABILITIES: We use the word RANDOM whenever the data $X, Y, Z, \ldots$ we are studying are produced by such an intricate mechanism that all we know about them is

(1) The range of possible values that $X, Y, Z, \ldots$ may take. This range is usually referred to as the SAMPLE SPACE and denoted by the symbol Ω.
(2) Certain positive numbers called PROBABILITIES which numerically express our "confidence" that $X, Y, Z, \ldots$ fall in chosen subsets of the sample space Ω.

ELEMENTARY OUTCOME, SAMPLE POINT: An individual outcome of the experiment $\mathcal{E}$ is usually referred to as an ELEMENTARY OUTCOME or SAMPLE POINT. Mathematically this is just an element of the sample space Ω.

EVENT: Mathematically an EVENT is just a subset of Ω. We say that $\mathcal{E}$ "resulted in the event A" or that "A has occurred" if the outcome falls in the subset A.

FIELD OF EVENTS: The collection of events associated with our experiment $\mathcal{E}$ is usually denoted by $\mathcal{F}$. In other words, $\mathcal{F}$ denotes the collection of subsets of the sample space Ω that are of special interest in our study. For mathematical reasons $\mathcal{F}$ is assumed to be closed under the set operations of intersection, union and complementation. The two subsets ϕ and Ω are always included in $\mathcal{F}$.

PROBABILITY MEASURE: Our experiment $\mathcal{E}$ associates to each event A of F a number $P[A]$ in the interval $[0, 1]$ which is reflects our confidence that the outcome falls in A. We refer to $P[a]$ as the "probability of A". Note that we should have

$P[\Omega] = 1$ and that if A and B are mutually exclusive events then

$$P[A \cup B] = P[A] + P[B]$$

A set function with these properties is usually referred to as a PROBABILITY MEASURE.

EXPECTATION OF A RANDOM VARIABLE: Any function of the outcome of our experiment can be referred to as a RANDOM VARIABLE. Mathematically, a random variable is simply a function on the sample space. If the events $A_1, A_2, \ldots, A_k$ are mutually exclusive and decompose Ω , and the random variable X takes the value x_i when A_i occurs then the expression

$$E[X] = x_1 P[A_1] + x_2 P[A_2] + \cdots + x_k P[A_k]$$

is referred to as the EXPECTATION OF X. If we repeat $\mathcal{E}$ a very large number of times, and average out the successive values of X we get, then we should **expect** the resulting average to be close to $E[X]$.

CONDITIONAL PROBABILITY: If **A** and **B** are events the ratio

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

is usually referred to as the CONDITIONAL PROBABILITY OF **A** GIVEN **B** The concept arises as follows. Given the event B we can construct a new experiment $\mathcal{E}_B$ by carrying out $\mathcal{E}$ and recording its outcome **only** when it falls in **B**. We can argue that the probability of **A** under $\mathcal{E}_B$ will is $\frac{P[A \cap B]}{P[B]}$ where $P[A \cap B]$ and $P[B]$ are the probabilities of $\mathbf{A} \cap \mathbf{B}$ and **B** under $\mathcal{E}$. We shall refer to $\mathcal{E}_B$ as $\mathcal{E}$ CRIPPLED by **B**.

CONDITIONAL EXPECTATION OF A RANDOM VARIABLE: Given an event B, if we carry out the crippled experiment $\mathcal{E}_B$ instead of $\mathcal{E}$, then all the probabilities change and so do all expectations. If X is a random variable and the events $A_1, A_2, \ldots, A_k$ decompose Ω as before then expression

$$E[X|B] = x_1 P[A_1|B] + x_2 P[A_2|B] + \cdots + x_k P[A_k|B]$$

gives the expected value of X under $\mathcal{E}_B$. We refer to it as the CONDITIONAL EXPECTATION OF X GIVEN B.

DEPENDENCE: The random variable Y is said to be DEPENDENT upon the random variable X if and only if Y is a function of X. Similarly we say that Y is dependent upon $X_1, X_2, \ldots, X_n$ if for some function $f(x_1, x_2, \ldots, x_n)$ we have

$$Y = f(X_1, X_2, \ldots, X_n)$$

INDEPENDENCE: In probability theory, "independence" is not the negation of "dependence" We say that Y is " independent" of X only if knowing the value of X "doesn't change our uncertainty" about Y. More precisely, if we cripple our experiment $\mathcal{E}$ by any of the events $[X = a]$ the probabilities of all the events $[Y = b]$ do not change. Mathematically this is translated in the conditions that for all choices of a and b

$$P[Y = b | X = a] = P[Y = b]$$

this simply means that

$$P[(Y = b) \cap (X = a)] = P[X = a] \times P[Y = b]$$