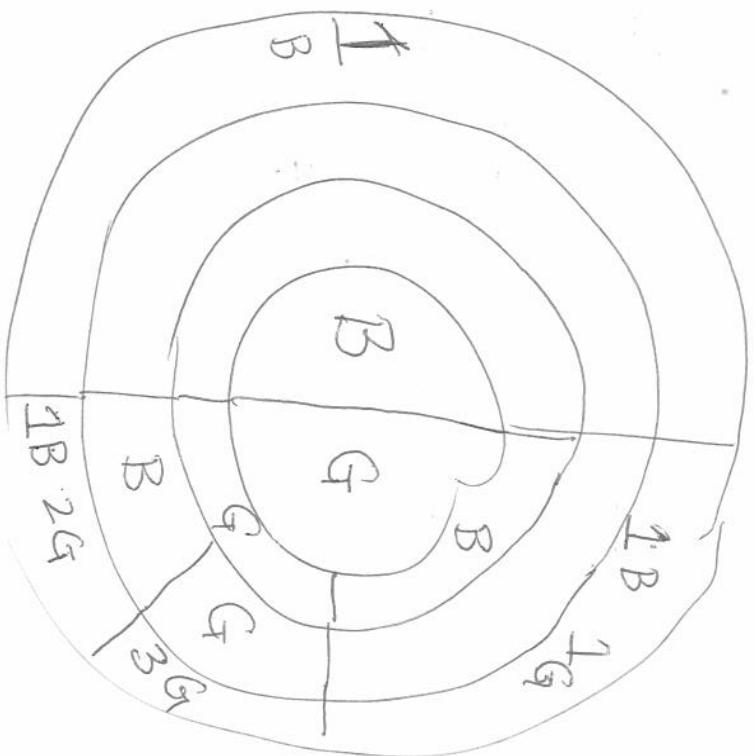


Couples in a given country want to have boys so they decide that they decide if they have a boy or if they have three girls in a row then they will stop having children.

What is the expected number of boys and girls on average for 10,000 couples?:

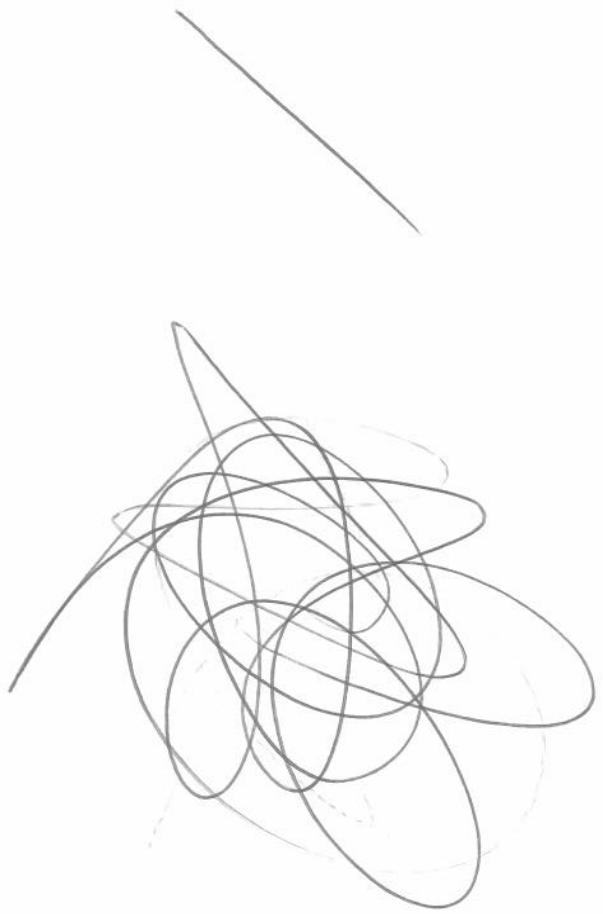
- 2 (a) more than half boys
- 4 (b) more than half girls
- 0 (c) same number of boys and girls
- 6 (d) don't know/don't care



$$E(\text{Boys}) = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \cdot 1 + \frac{1}{8} \cdot 0 = \frac{7}{8}$$

$$E(\text{Girls}) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 2 \cdot \frac{1}{8} = \frac{7}{8}$$

Sequences which have
very low or high probability
also have less information



the outcome
of a 50/50
coin toss
has 1 bit
of information.

Information Theory Definitions

Definition: The Entropy of a random variable X

$$H(X) = \sum_a P[X = a] \log_2 \left(\frac{1}{P[X = a]} \right)$$

Expected value of the entropy of X

Definition: The entropy of two random variables X and Y .

$$H(X, Y) = \sum_{a,b} P[X = a \ \& \ Y = b] \log_2 \left(\frac{1}{P[X = a \ \& \ Y = b]} \right)$$

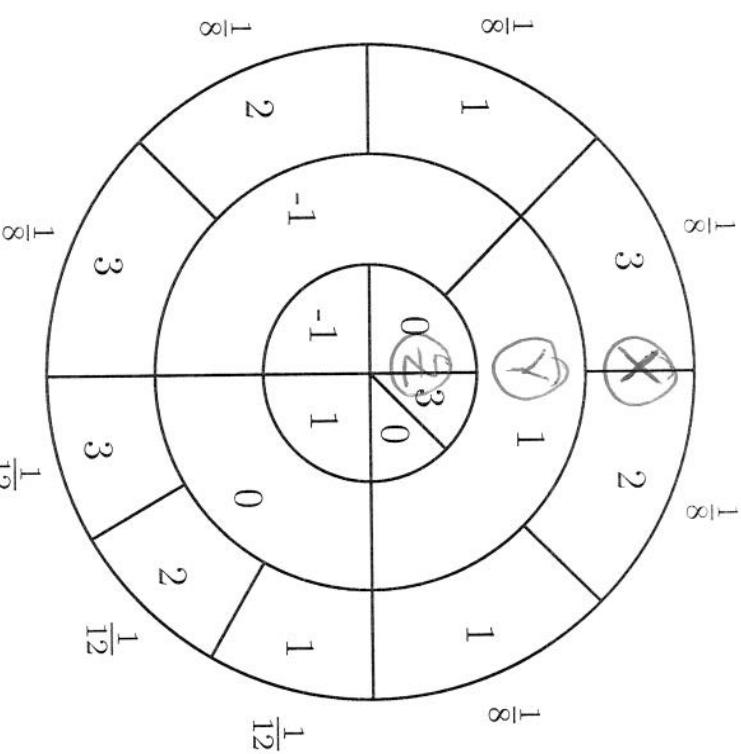
(a) Calculate $H[X]$.

(b) Calculate the expected number of binary registers needed to store Z .

(c) Calculate the uncertainty of Z given that $X = 0$.

(d) Calculate $H[X|Y, Z]$.

(e) Calculate $H[Z|Y]$.



$$\mathcal{H}(X) = \frac{1}{3} \log_2 3 + \frac{1}{3} \log_2 3 + \frac{1}{3} \log_2 3 = \log_2 3$$

$$\mathcal{H}(Z) = \frac{3}{8} \log_2 \left(\frac{8}{3} \right) + \frac{1}{4} \log_2 4 + \frac{1}{4} \log_2 4 + \frac{1}{8} \log_2 8$$