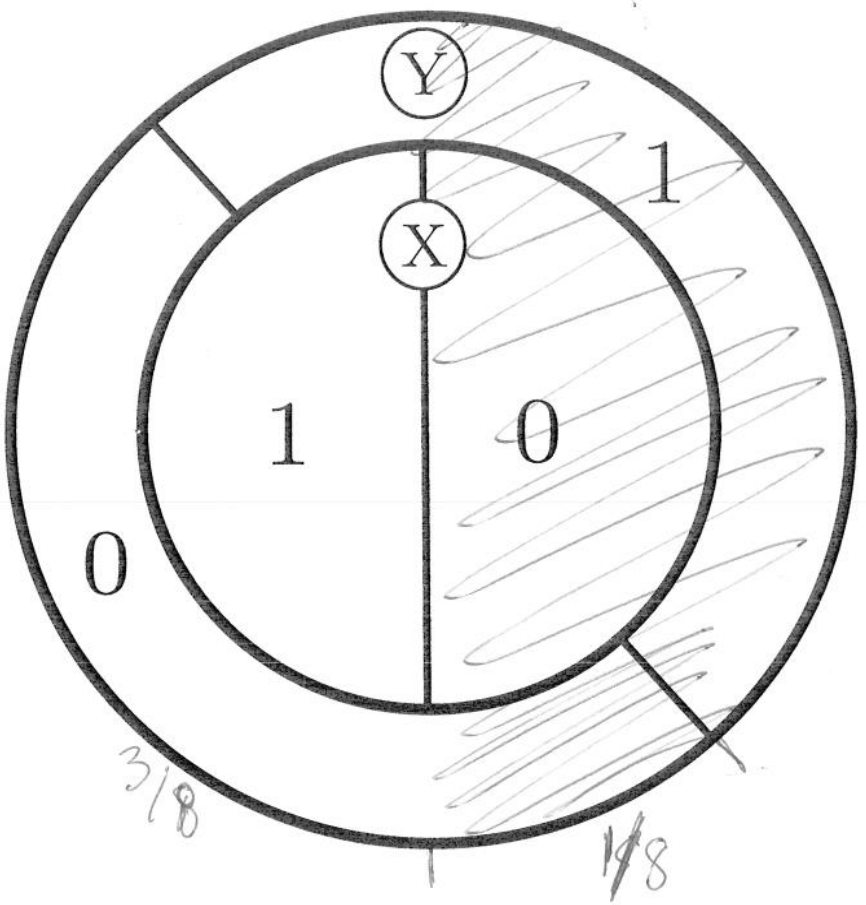
Wheel = Random variable



$P(X=1) = 1/2$

$P(Y=1) = 1/2$

$P(X=0) = 1/2$

$P(Y=0) = 1/2$

$P(X=0 \& Y=0) = 1/8$

$P(X=1 \& Y=1) = 1/8$

$P(X=1 \& Y=0) = 3/8$

$P(X=0 \& Y=1) = 3/8$

$$P(Y=0 \mid X=1) = \frac{P(Y=0 \& X=1)}{P(X=1)} = \frac{3/8}{1/2} = 3/4$$

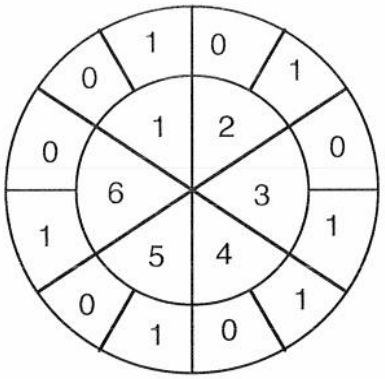
$P(Y=0 \mid X=1) = 1/4$

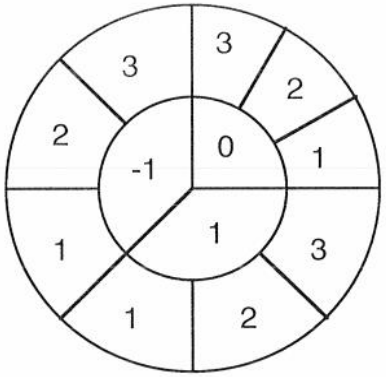
X is independent of Y if $P(X = a|Y = b) = P(X = a)$

or $P(X = a \text{ and } Y = b) = P(X = a)P(Y = b)$

or knowing the value of Y does not change the probabilities of X

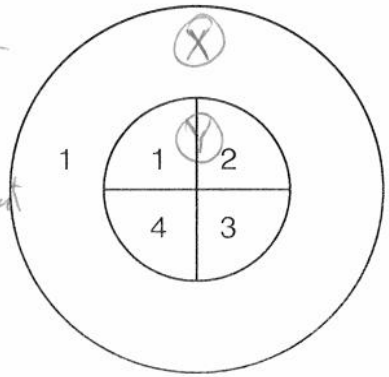
If X is independent of Y, then Y is independent of X.

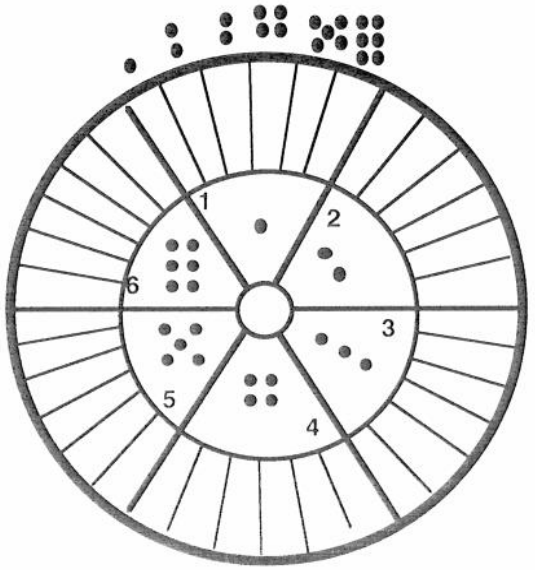


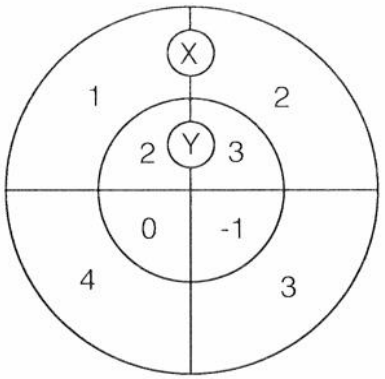


X is dependent on Y

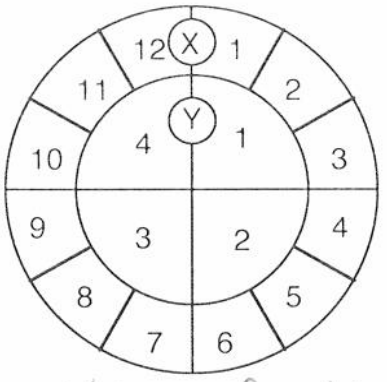
X is independent of Y





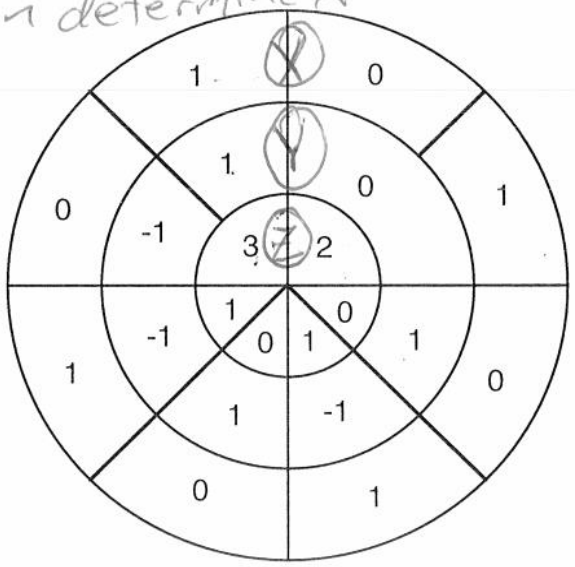


X is a function of Y because if I know Y then I can determine X



Y is a function of X
but X is not a function of Y
Y is dependent on X

Z is dep on X & Y
Y is dep on X & Z
X is not dep on Y & Z



X is not dependent on Y
X is not dependent Z
Y is not dep. on X
Y is not dep on Z
Z is not dep on X
Z is not dep on Y

X is dependent on Y if X is a function of Y

that is, knowing the value of Y determines the value of X

DEPENDENCE: The random variable Y is said to be DEPENDENT upon the random variable X if and only if Y is a function of X . Similarly we say that Y is dependent upon X1 , X2 , . . . , Xn if for some function $f(x_1, x_2, \ldots, x_n)$ we have

$$Y = f(X_1, X_2, \ldots, X_n)$$

INDEPENDENCE: In probability theory, "independence" is not the negation of "dependence" We say that X is "independent" of Y only if knowing the value of Y "doesn't change our uncertainty" about X. More precisely, if we cripple our experiment E by any of the events [Y = b] the probabilities of all the events [X = a] do not change. Mathematically this is translated in the conditions that for all choices of a and b

$$\frac{P(X=a \,\&\, Y=b)}{P(Y=b)} = P(X = a|Y = b) = P(X = a)$$

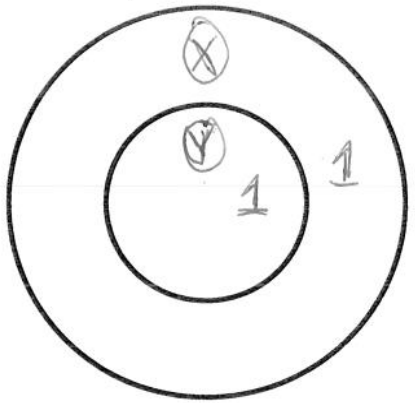
this simply means that

$$P(X = a \text{ and } Y = b) = P(X = a)P(Y = b)$$

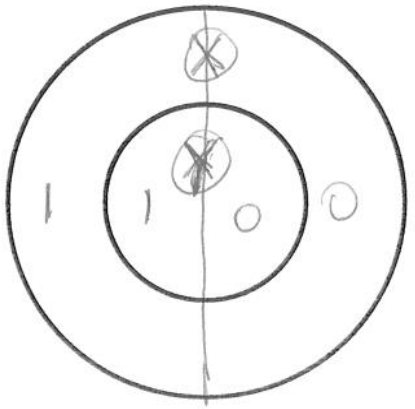
"X is dependent on Y" and "X is independent of Y" are not opposite statements of each other, rather they are on opposite sides of a spectrum of possibilities.

"X is not dependent on Y" does not mean "X is independent of Y"

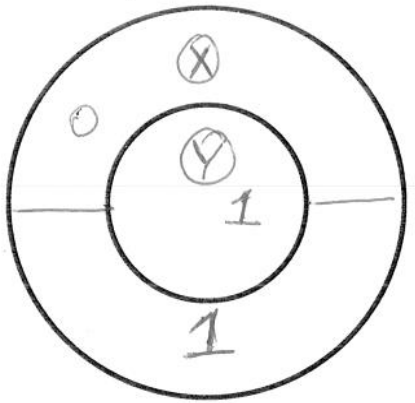
X is independent of Y
X is dependent on Y
Y is dependent on X



X is not independent of Y
X is dependent on Y
Y is dependent on X

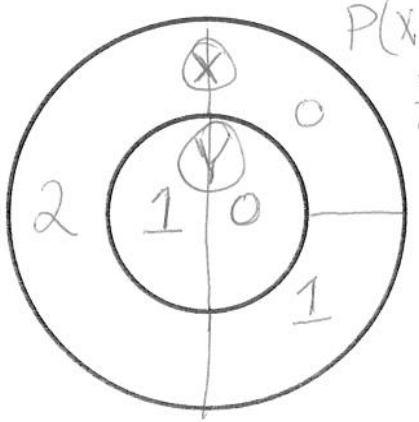


X is independent of Y
X is not dependent on Y
Y is dependent on X

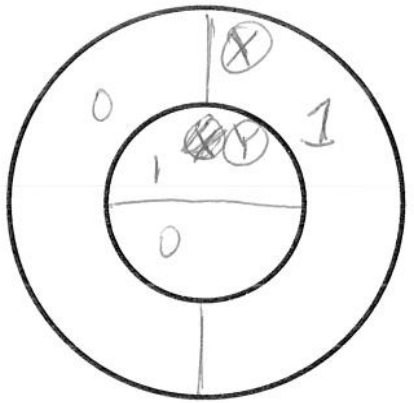


X is not independent of Y
X is not dependent on Y
Y is dependent on X

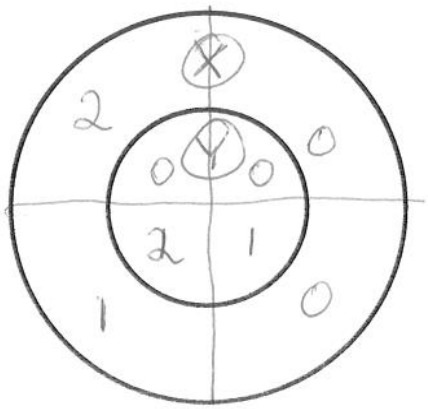
$P(X=2|Y=1)=1$
$P(X=2)=1/2$

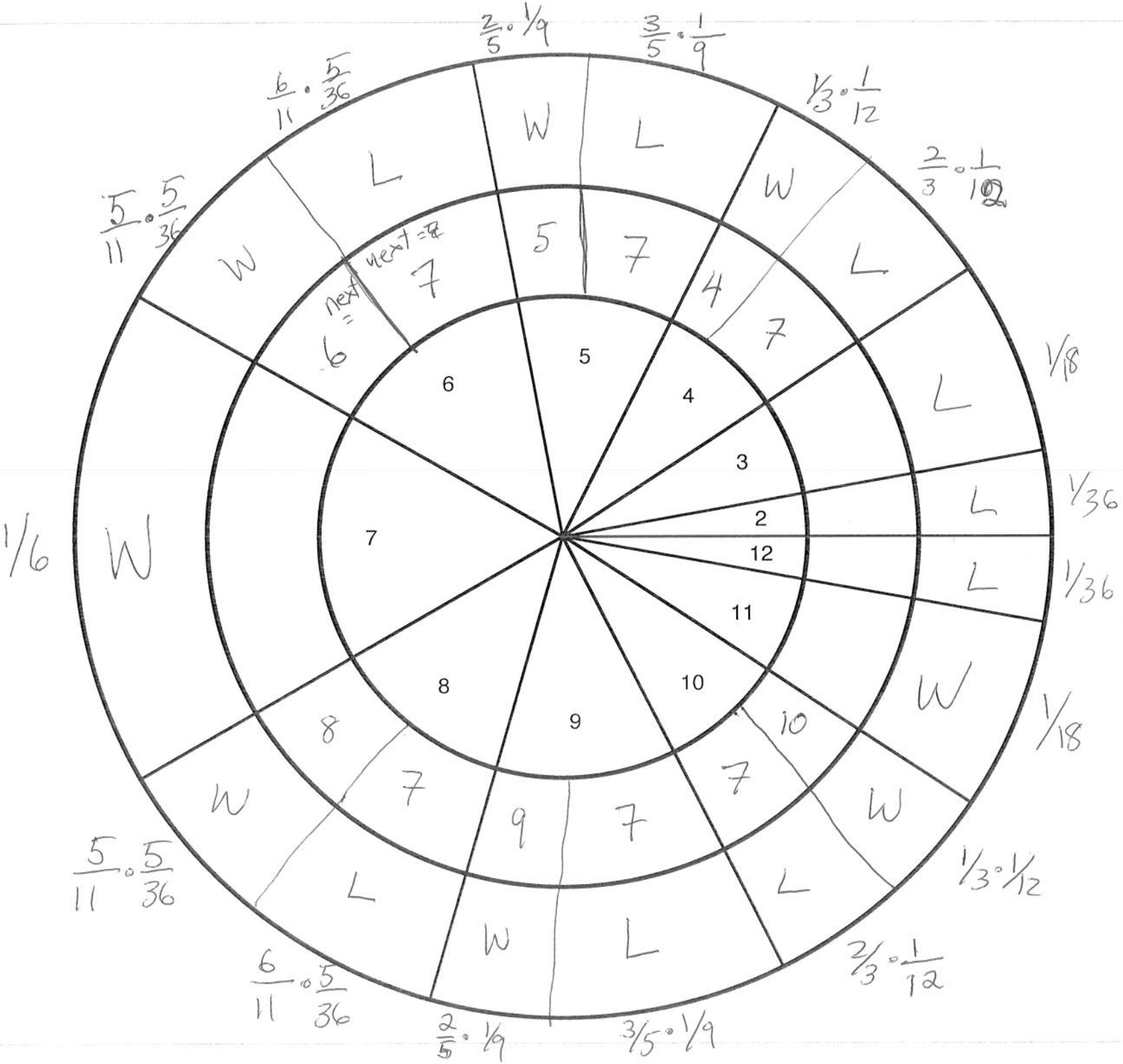


X is independent of Y
X is not dependent on Y
Y is not dependent on X



X is not independent of Y
$P(Y=2|X=1)=1$ $P(Y=2)=1/4$
X is not dependent on Y ✓ check Y=0
Y is not dependent on X ✓ check X=0




2/5 · 1/9
3/5 · 1/9
1/3 · 1/12
2/3 · 1/12
1/18
1/36
1/36
1/18
1/3 · 1/12
2/3 · 1/12
3/5 · 1/9
2/5 · 1/9
6/11 · 5/36
5/11 · 5/36
1/6
5/11 · 5/36
6/11 · 5/36
W
L
next = 6
next = 7
1
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3
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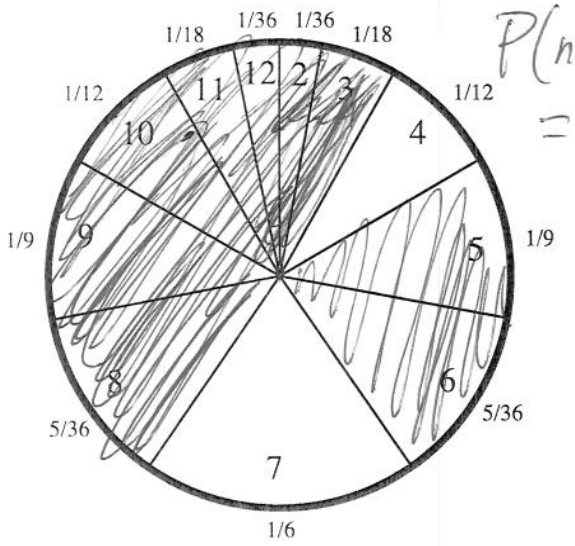
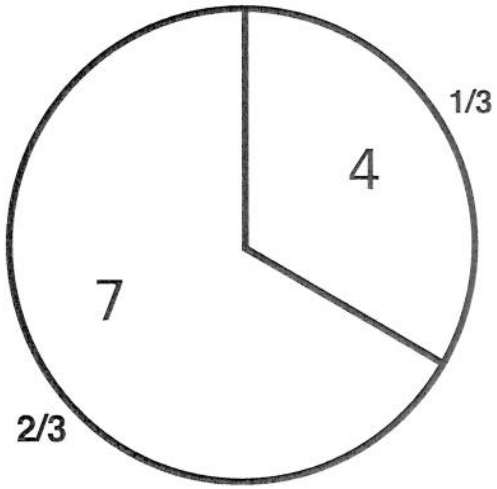
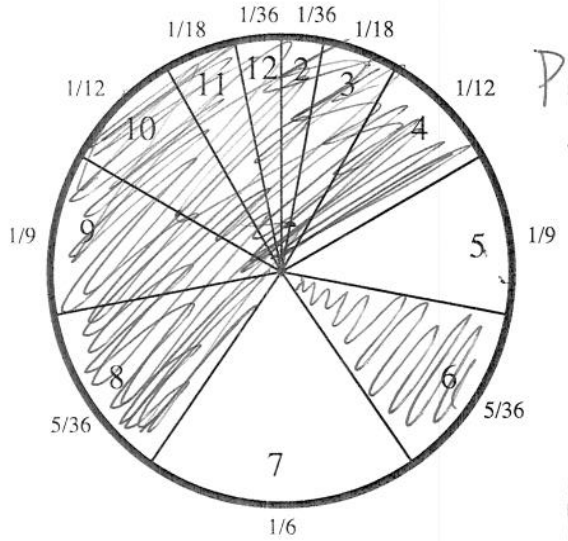
11
12

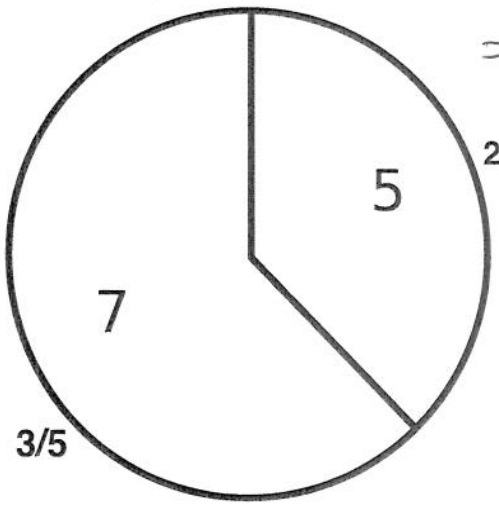


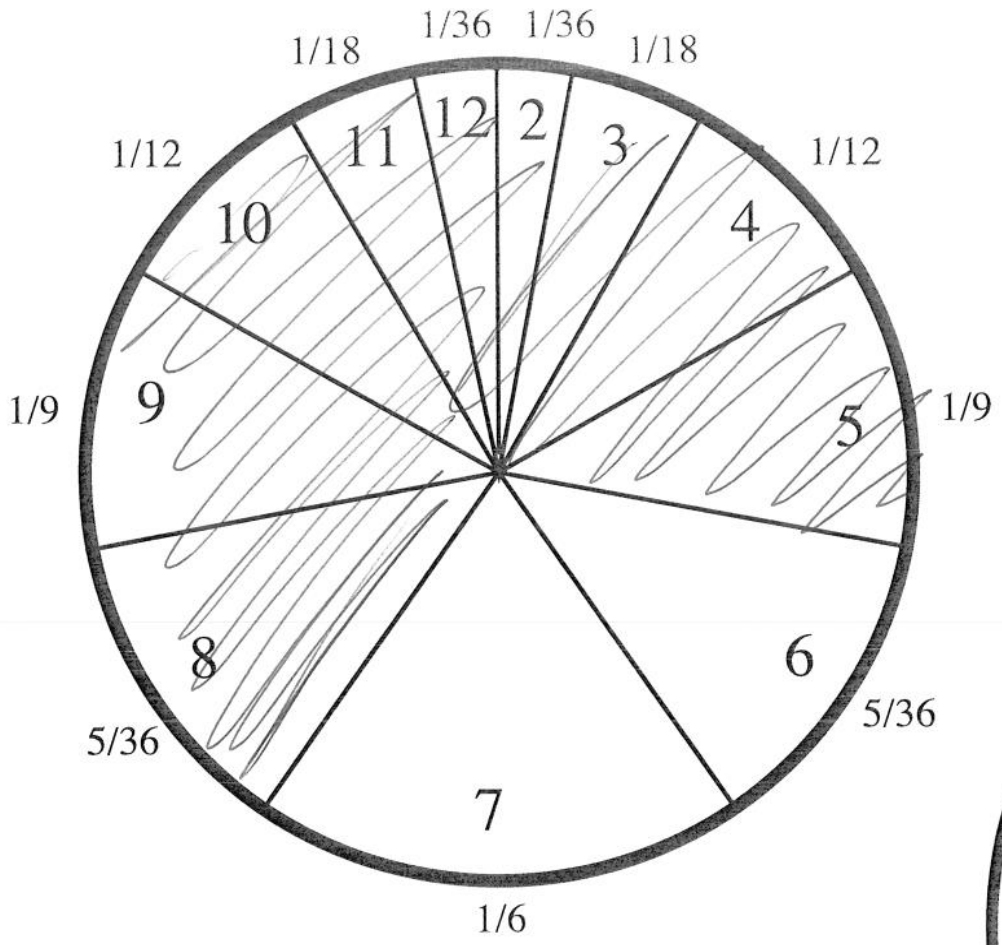
$$P(\text{next}=4 \mid \text{next}=4 \text{ OR } 7) = \frac{P(\text{next}=4 \text{ \& next}=4 \text{ OR } 7)}{P(\text{next}=4 \text{ OR } 7)} = \frac{3/36}{9/36} = 1/3$$





$$P(\text{next}=5 \mid \text{next}=5 \text{ OR } 7) = \frac{P(\text{next}=5 \text{ \& next}=5 \text{ OR } 7)}{P(\text{next}=5 \text{ OR } 7)} = \frac{1/9}{1/6 + 1/9} = \frac{4/36}{10/36} = \frac{4}{10} = \frac{2}{5}$$





$P(\text{next roll} = 6 \mid \text{next roll} = 6 \text{ OR } 7)$

$= \dfrac{P(\text{next roll} = 6 \;\&\; \text{next roll} = 6 \text{ OR } 7)}{P(\text{next roll} = 6 \text{ OR } 7)}$

$= \dfrac{5/36}{11/36} = \dfrac{5}{11}$

$P(\text{next roll} = 6 \text{ OR } 7) = 11/36 = 5/36 + 6/36$

$P(\text{next roll} = 6 \;\&\; \text{next roll} = 6 \text{ OR } 7) = 5/36$