FINAL EXAM (PART II - QUESTION 6 THROUGH 10) OF MATH 5020

ASSIGNED: FEBRUARY 16, 2004 - DUE: MARCH 1, 2004

There are a few instructions for this exam that I want to give in advance:

- I am not expecting you to spend a lot of time explaining the answer to the following questions, but I do expect that you write a sentence or two which tells me that you know what you are doing (i.e. DON'T give me just the answer to the following problems).
- On the previous exam I asked you to work alone and this instruction was only followed to a certain degree. On this exam you may work with another person, however if you choose to speak with someone else about this exam they must do a different problem as you. Please write the name and question number of any person you discuss a problem with on the exam. When you discuss a question, please do not 'give' the solution to someone. I have the right to give a 0 if these instructions are not followed.
- You may ask me questions although I would like you to ask them in the FORUM, this way everyone has the chance to read the same instructions/information.
- Do one of problems 1 and 2 and one of problems 3, 4 or 5.

(1) Let $A(q)$ represent the generating function $a_0 + a_1 q + a_2 q^2 + a_3 q^3 + a_4 q^4 + \cdots$. Find the coefficient of q^n in the following expressions.

(a)
$$(A(q) + A(-q))/2$$

(b)
$$(A(q) - A(-q))/2$$

(c)
$$A(5q)$$

(d)
$$A(q^2)$$

(e)
$$\frac{1}{k!}\frac{d^k}{dq^k}A(q)$$

(2) Find the coefficient of q^7 in the following generating functions

(a)
$$\frac{1-q^3}{1-q}\frac{1}{1-5q}$$

(b)
$$\frac{1-q^{12}}{1-q}(1+q+q^4+q^5)$$

(c)
$$\frac{q^3}{(1-3q)^5}$$

(d)

$$q(1+4q^2)^7$$

(e)

$$\frac{1+q^3}{1+q}\frac{1}{(1-q)^{10}}$$

(3) Using identity (9) and (10) from the handout "Some connections between Algebraic Expressions and Sequences : Part II" we know that $L(q) = (1+2q)F(q)$ where $L(q) = \sum_{n\geq 0} L_{n+1}q^n$ and $F(q) = \sum_{n\geq 0} F_{n+1}q^n$. Use this to show identity:

$$L_n - 2L_{n-1} + 4L_{n-2} - \cdots + (-1)^{n-1}2^{n-1}L_1 = F_n$$

(4) Use the identity $\frac{q+q^2}{(1-q)^3} = \sum_{n\geq 0} n^2q^n$ and $\frac{1}{(1-q)^3} = \sum_{n\geq 0}\binom{n+2}{2}$ to show the following identity:

$$n^2 - (n-1)^2 + (n-2)^2 - (n-3)^2 + \cdots + (-1)^{n-1}1^2 = \binom{n+1}{2}$$

(5) How many ways are there of choosing 60 marbles from a collection of 50 red, 35 blue and 15 green?