

## LOTTERY 6/49

VERSION 1 : JANUARY 31, 2003

The probability of having a winning ticket with property  $X$  will be equal to the fraction of tickets with property  $X$ . That is,

$$\text{the probability of a win with property } X = \frac{\text{the number of tickets which have property } X}{\text{The total number of possible 6/49 tickets}}$$

The total number of 6/49 tickets is equal to  $\binom{49}{6}$ .

To calculate these lottery probabilities, “property  $X$ ” is one of: “all 6 winning numbers,” “5 of 6 winning numbers and the bonus,” “5 of 6 winning numbers and not the bonus,” “4 of 6 winning numbers,” and “3 of 6 winning numbers.”

The number of tickets with  $k$  numbers chosen from the winning numbers and  $6 - k$  numbers chosen from the non-winning numbers will be  $\binom{6}{k} \binom{43}{6-k}$ . There is an exception to this in the condition “5 of 6 winning numbers and not the bonus” since the remaining number must be chosen from any of the remaining 43 numbers *except* the bonus and so the number of tickets with this property will be  $\binom{6}{5} \binom{42}{1}$ . The number of tickets which have 5 of 6 winning numbers and the bonus will be  $\binom{6}{5} \binom{1}{1}$  since there is exactly one bonus number.

This gives us that the probabilities are calculated as follows:

$$\begin{aligned} \text{probability of having all 6 winning} &= \frac{\binom{6}{6} \binom{43}{0}}{\binom{49}{6}} = \frac{1}{\frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{1}{13983816} \\ \text{probability of having 5 of 6 winning numbers and the bonus} &= \frac{\binom{6}{5} \binom{1}{1}}{\binom{49}{6}} = \frac{6}{13983816} = \frac{1}{2330636} \\ \text{probability of having 5 of 6 winning numbers and not the bonus} &= \frac{\binom{6}{5} \binom{42}{1}}{\binom{49}{6}} = \frac{6 \cdot 42}{13983816} \approx \frac{1}{55491} \\ \text{probability of having 4 of 6 winning numbers} &= \frac{\binom{6}{4} \binom{43}{2}}{\binom{49}{6}} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{43 \cdot 42}{2 \cdot 1} \approx \frac{1}{1033} \\ \text{probability of having 3 of 6 winning numbers} &= \frac{\binom{6}{3} \binom{43}{3}}{\binom{49}{6}} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \cdot \frac{43 \cdot 42}{2 \cdot 1} \approx \frac{1}{57} \end{aligned}$$