CALCULATING THE PROBABILITIES OF WINNING LOTTO 6/49

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The probability of event occuring is a measure of the likelihood that event will occur and is always a value between 0 and 1 (0 means that the event never happens and 1 means that the event happens 100% of the time). The probability of having a ticket with a given property will be equal to the fraction of tickets with that property. That is,

$$\text{the probability of a win with a certain property } = \frac{\text{the number of tickets which have the property}}{\text{the total number of possible 6/49 tickets}}$$

The number of ways of selecting r items from a set of n items is denoted by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ where $n!$ represents $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$.

A 6/49 ticket consists of 6 of numbers between 1 and 49 and since the number of selections of 6 numbers from a set of 49 different numbers is $\binom{49}{6} = 13{,}983{,}816$, this is equal to the total number of 6/49 tickets. Now to find the probability of winning in each of the prize categories we need only determine how many tickets have a given property.

Winning without the bonus

Of the 6 winning numbers, we must select k of them AND of the 43 non-winning numbers, we must select $(6-k)$ of them. Therefore there are $\binom{6}{k} \times \binom{43}{6-k}$ possible winning tickets matching k of the winning numbers.

There is an exception to this in the condition if we insist that the ticket not include the bonus number (e.g. the prize for "5 of 6 winning numbers and not the bonus" because the tickets with "5 of 6 winning numbers and the bonus" win a bigger prize). In this case the number of tickets which include k winning numbers AND $6-k$ of the 42 non-winning numbers which are not the bonus will be $\binom{6}{k} \times \binom{42}{6-k}$.

Winning with the bonus

The number of tickets which have k winning numbers and the bonus can be found by choosing k of the 6 winning numbers AND the bonus number AND choosing $5-k$ of the 42 non-winning/non-bonus numbers. This means that there are $\binom{6}{k} \times \binom{42}{5-k}$ tickets which include exactly k winning numbers and the bonus.

This gives us that the probabilities are calculated as follows:

$$\text{probability of having all 6 winning} = \frac{\binom{6}{6}\binom{43}{0}}{\binom{49}{6}} = \frac{1}{\frac{49\cdot 48\cdot 47\cdot 46\cdot 45\cdot 44}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}} = \frac{1}{13983816}$$

$$\text{probability of having 5 of 6 winning numbers and the bonus} = \frac{\binom{6}{5}\binom{1}{1}}{\binom{49}{6}} = \frac{6}{13983816} = \frac{1}{2330636}$$

$$\text{probability of having 5 of 6 winning numbers and not the bonus} = \frac{\binom{6}{5}\binom{42}{1}}{\binom{49}{6}} = \frac{6\cdot 42}{13983816} \approx \frac{1}{55491}$$

$$\text{probability of having 4 of 6 winning numbers} = \frac{\binom{6}{4}\binom{43}{2}}{\binom{49}{6}} = \frac{\frac{6\cdot 5\cdot 4\cdot 3}{4\cdot 3\cdot 2\cdot 1}\cdot\frac{43\cdot 42}{2\cdot 1}}{13983816} \approx \frac{1}{1033}$$

$$\text{probability of having 3 of 6 winning numbers} = \frac{\binom{6}{3}\binom{43}{3}}{\binom{49}{6}} = \frac{\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1}\cdot\frac{43\cdot 42}{2\cdot 1}}{13983816} \approx \frac{1}{57}$$