SOME CONNECTIONS BETWEEN ALGEBRAIC EXPRESSIONS AND SEQUENCES : PART IV

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Match the generating function with the formula for the sequence. With a computer it is usually easy to figure out the generating function with the sequence by using the command `taylor` in Maple or the OLEIS if the entry happens to list the generating function. If you don't have a computer then you will need to use your algebra skills and it is usually easier to go from the sequence to the generating function than the reverse.

(1) $a_n = 3 \cdot 2^n$
(2) $a_n = \binom{n}{2} 2^n$
(3) $a_n = \binom{3}{n} 2^n$
(4) $a_0 = 17$, $a_1 = 3$, $a_2 = -1$, $a_3 = 12$, $a_4 = 1$, $a_5 = -3$, $a_6 = 1$, $a_n = 0$ for $n > 6$.
(5) $a_0 = 1$, $a_1 = 3$, $a_2 = 5$, $a_3 = 6$, $a_4 = 5$, $a_5 = 3$, $a_6 = 1$, $a_n = 0$ for $n > 6$.
(6) $a_0 = a_k = 1$, $a_r = 2$ for $1 < r < k$, $a_n = 0$ for $n > k$
(7) $a_n = 2n$
(8) $a_n = n^2$
(9) $a_n = n^3$
(10) $a_n = n^4$
(11) $a_n = n(n-1)$
(12) $a_n = n(n-1)(n-2)$
(13) $a_n = n(n-1)\cdots(n-k+1)$
(14) $a_n = n(n+1)$
(15) $a_n = n(n+1)(n+2)$
(16) $a_n = n(n+1)\cdots(n+k-1)$
(17) $a_n = n^2 2^n$
(18) $a_n = (n+1)\binom{4}{n}$

(a) $3/(1-2q)$
(b) $2q/(1-q)^2$
(c) $(1-q^2)(1-q^k)/(1-q)^2$
(d) $2/(1-q)^3$
(e) $(1+2q)^3$
(f) $(2q+4q^2)/(1-2q)^3$
(g) $(1+5q)(1+q)^3$
(h) $2q^2/(1-q)^3$
(i) $q^2/(1-2q)^3$
(j) $(q+q^2)/(1-q)^3$
(k) $(1-q^2)(1-q^3)(1-q^4)/(1-q)^3$
(l) $6q^3/(1-q)^4$
(m) $6/(1-q)^4$
(n) $(q+4q^2+q^3)/(1-q)^4$
(o) $(q+11q^2+11q^3+q^4)/(1-q)^5$
(p) $17+3q-q^2+12q^3+q^4-3q^5+q^6$
(q) $k!q^{k-1}/(1-q)^{k+1}$
(r) $k!/(1-q)^{k+1}$