PARTITION GENERATING FUNCTIONS : PART I

MARCH 8, 2004

Match the description of the set of partitions with its generating function. Recall that a *partition* of n is a sum $\lambda_1 + \lambda_2 + \cdots + \lambda_r = n$. The order of the sum doesn't matter so to avoid confusion we assume that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r$. The λ_i are called the *parts* of the partition. r here is the number of parts of the partition or the *length* of the partition. The *sizes* of the parts are the values λ_i. The *size* of the partition is the sum of the sizes of all the parts (in this case n). Parts are called *distinct* if they are not equal to each other. The *number of parts of a given size* refers to the number of times that a value appears as a part.

Note: There 14 generating functions and 15 descriptions listed below because two of the descriptions have the same generating function.

(1) the number of partitions of n
(2) the number of partitions of n into exactly k parts
(3) the number of partitions of n with parts of size k only
(4) the number of partitions of n with parts of size less than or equal to k
(5) the number of partitions of n with distinct parts
(6) the number of partitions of n with odd parts
(7) the number of partitions of n with distinct odd parts
(8) the number of partitions of n with even parts
(9) the number of partitions of n with distinct even parts
(10) the number of partitions of n into parts congruent to 1 or 4 modulo 5
(11) the number of partitions of n with at most 4 parts of any given size
(12) the number of partitions of n with (for each i) the number of size i is less than i.
(13) the number of partitions of n and for each i, if there is a part of size i then it occurs an odd number of times.
(14) the number of partitions of n and for each i, the parts of size i occur an even number of times.
(15) the number of partitions of n with only odd parts and the number of parts of any given size is even.

(a)

$$\prod_{i \geq 1} \frac{1}{1 - q^{2i-1}}$$

(b)

$$\frac{1}{1 - q^k}$$

(c)

$$\prod_{i=1}^{k} \frac{1}{1 - q^i}$$

(d)

$$\prod_{i\geq 1}\frac{1}{1-q^{4i-2}}$$

(e)

$$\prod_{i\geq 1}(1+q^i)$$

(f)

$$\prod_{i\geq 1}\left(1+\frac{q^i}{1-q^{2i}}\right)$$

(g)

$$\prod_{i\geq 0}\frac{1}{(1-q^{5i+1})(1-q^{5i+4})}$$

(h)

$$\prod_{i\geq 1}\frac{1}{1-q^i}$$

(i)

$$\prod_{i\geq 1}(1+q^{2i-1})$$

(j)

$$q^k\prod_{i=1}^{k}\frac{1}{1-q^i}$$

(k)

$$\prod_{i\geq 1}(1+q^{2i})$$

(l)

$$\prod_{i\geq 1}\frac{1}{1-q^{2i}}$$

(m)

$$\prod_{i\geq 1}\frac{1-q^{5i}}{1-q^i}$$

(n)

$$\prod_{i\geq 1}\frac{1-q^{i^2}}{1-q^i}$$