

## WORKSHEET I: SEQUENCES AND SETS OF OBJECTS

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Write the first 6-8 terms of the following sequences. Assume that the sequences start at  $n = 0$ , write a formula for  $a_n$  if possible (HINT: the empty word is a word of length 0). The OLEIS sequence number can be found by going to the web site 'The On-Line Encyclopedia of Integer Sequences' and entering the first terms which you calculated. It may well be that the sequence that you entered is not in the database. Your next step will be to calculate more terms and try to arrive at a formula for  $a_n$ . Again, this might not be possible. Speak to me because we might be able to solve this problem together. :

- (1) The number of solutions to  $x_1 + x_2 + x_3 + x_4 = n$  with  $x_i \geq 0$  with  $x_4$  odd and  $x_3$  even. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (2) The number of solutions to  $x_1 + x_2 + x_3 + x_4 = n$  with  $i \geq x_i \geq 0$  with  $x_4$  even and  $x_3$  even. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (3) The number of words of length  $n$  created with the letters  $a$  and  $b$  such that no  $a$  is adjacent to a  $b$ . \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (4) The number of words of length  $n$  created with the letters  $a$  and  $b$  such that every  $a$  is separated by at least two  $b$ 's. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (5) The number of words of length  $n$  created with the letters  $a$  and  $b$  such that every  $a$  is separated by at least three  $b$ 's. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (6) The number of words of length  $n$  created with the letters  $a, b, c$  with at least half of the letters are  $a$ 's. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (7) The number of words of length  $n$  created with the letters  $a, b, c$  with no consecutive letters being equal. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (8) The number of words of length  $n$  created with the letters  $a, b, c$  with all  $c$ 's appearing after all of the  $b$ 's. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (9) The number of words of length  $n$  created with the letters  $a, b, c$  with at least as many  $a$ 's as  $b$ 's and at least as many  $b$ 's as  $c$ 's. \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (10) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $b$  adjacent to at least one  $c$ . \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (11) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $b$  adjacent to at least one  $c$  and one  $a$ . \_\_\_\_\_  
Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_

- (12) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $c$  not adjacent to any  $as$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (13) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $b$  occurring in groups of two or more. \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (14) The number of words of length  $n$  created with the letters  $a, b, c$  with no adjacent  $bs$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (15) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $a$  and every  $b$  adjacent to at least one  $c$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (16) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $b$  separated from every  $c$  by at least one  $a$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (17) The number of words of length  $n$  created with the letters  $a, b, c$  with every  $b$  separated from every  $c$  by at least two  $as$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (18) The number of words of length  $n$  created with the letters  $a, b, c$  with more  $cs$  than either  $as$  or  $bs$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (19) The number of words of length  $n$  created with the letters  $a, b, c$  with more  $cs$  than the number of  $as$  and  $bs$  put together. \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_
- (20) The number of words of length  $n$  created with the letters  $a, b, c$  with more  $cs$  than  $bs$  and more  $bs$  than  $as$ . \_\_\_\_\_  
 Formula?  $a_n =$  \_\_\_\_\_ OLEIS sequence number \_\_\_\_\_