

### WORKSHEET III: ALGEBRAIC EXPRESSIONS AND SEQUENCES

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Let  $A(q) = a_0 + a_1q + a_2q^2 + a_3q^3 + \dots$  and  $B(q) = b_0 + b_1q + b_2q^2 + b_3q^3 + \dots$  where the integers  $a_i$  represent a sequence and  $b_i$  represent another sequence and  $q$  is a variable. That is, the coefficient of  $q^n$  in  $A(q)$  is  $a_n$  and the coefficient of  $q^n$  in  $B(q)$  is  $b_n$ . Recall that  $1/(1-q) = 1 + q + q^2 + q^3 + q^4 + \dots$  and by taking the derivative of this equation we can show that

$$1/(1-q)^2 = \frac{d}{dq}(1/(1-q)) = \frac{d}{dq}(1 + q + q^2 + q^3 + q^4 + \dots) = 1 + 2q + 3q^2 + 4q^3 + \dots$$

What is the coefficient of  $q^n$  in the following expressions?

- (1)  $cA(q)$  ( $c$  is a constant here)
- (2)  $A(q) + B(q)$
- (3)  $q^m A(q)$
- (4)  $q^r A(q) + q^k B(q)$
- (5)  $A'(q)$
- (6)  $A''(q)$
- (7)  $A(q)B(q)$
- (8)  $A(q)^2$
- (9)  $A(q^2)$
- (10)  $A(q)/(1 - q^2)$
- (11)  $A(q)/(1 - q)^2$
- (12)  $A'(q)/(1 - q)$
- (13)  $(1 + q)^k A(q)$
- (14)  $A(q)/(1 - q)$
- (15)  $(1 + q + q^2)A(q)$
- (16)  $\frac{1 - q^k}{1 - q} A(q)$
- (17)  $A(q) + A(-q)$
- (18)  $A(q) - A(-q)$
- (19)  $A(q) + qA'(q)$
- (20)  $A(q) + 3qA'(q) + q^2A''(q)$
- (21)  $A(q) + 7qA'(q) + 6q^2A''(q) + q^3A'''(q)$
- (22)  $A'(q) + qA''(q)$
- (23)  $A(q)^3$
- (24)  $A(q)^4$
- (25)  $1/(1 - qA(q))$
- (26)  $A(qB(q))$