BIJECTION!

Let A_n be the set of words in the letters a and b of length n such that the word does not contain the pattern aba in adjacent letters.

What we will show that the set of words in $A_{n-1} \cup A_{n-2} \cup A_{n-4}$ is in bijection with A_n for $n \geq 4$.

Here is the map:
If $w \in A_{n-1}$ then let $\phi_n(w) = wb$.
If $w \in A_{n-2}$ and $w = ub$ (where u is a word), then $\phi_n(w) = ubba = wba$ and if $w = ua$, then $\phi_n(w) = uaaa = waa$.
If $w \in A_{n-4}$, then $\phi_n(w) = wbbaa$.

I assume at this point it is clear that for any word in A_{n-1}, A_{n-2} and A_{n-4} then $\phi_n(w)$ is in A_n because the the map does not introduce occurrences of aba in $\phi_n(w)$ if aba is not in w. We can also see that this map is injective since in each of the three cases that define this map, the endings of the images that we add on are all different because in the case of $w \in A_{n-1}$, $\phi_n(w)$ ends in b; if $w \in A_{n-1}$, then $\phi_n(w)$ ends in bba or aaa; if $w \in A_{n-4}$, then $\phi_n(w)$ ends in baa.

Now to show that the map is surjective, we start with any arbitrary word in A_n and show that there is an element v in $A_{n-1} \cup A_{n-2} \cup A_{n-4}$ such that $\phi_n(v) = w$. If w ends in b then $w = ub = \phi_n(u)$ where u is an element of A_{n-1}.

So now all the other words end in a so the last three letters are either bba, aaa or baa.

If w ends in bba then $w = ubba = \phi_n(ub)$ where ub is in A_{n-2}.

If w ends in aaa, then $w = uaaa = \phi_n(ua)$ where ua is in A_{n-2}.

Finally, if w ends in baa, then $w = ubaa$ and u doesn't end in a, so $w = vbbaa = \phi_n(v)$ for some word v in A_{n-4}. Therefore we have shown that ϕ_n is surjective and hence bijective.