

ALGEBRAIC EXPRESSIONS AND SEQUENCES

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Let $A(q) = a_0 + a_1q + a_2q^2 + a_3q^3 + \dots$ and $B(q) = b_0 + b_1q + b_2q^2 + b_3q^3 + \dots$ where the integers a_i represent a sequence and b_i represent another sequence and q is a variable. That is, the coefficient of q^n in $A(q)$ is a_n and the coefficient of q^n in $B(q)$ is b_n . Recall that $1/(1-q) = 1+q+q^2+q^3+q^4+\dots$ and by taking the derivative of this equation we can show that

$$1/(1-q)^2 = \frac{d}{dq}(1/(1-q)) = \frac{d}{dq}(1+q+q^2+q^3+q^4+\dots) = 1+2q+3q^2+4q^3+\dots$$

What is the coefficient of q^n in the following expressions?

- (1) $cA(q)$ (c is a constant here)
- (2) $A(q) + B(q)$
- (3) $q^7A(q)$
- (4) $q^2A(q) + q^3B(q)$
- (5) $A'(q)$
- (6) $\frac{d^k}{dq^k}A(q)$
- (7) $A'(q)B(q)$
- (8) $A(q)B(q)$
- (9) $A(q)^2$
- (10) $A(q^2)$
- (11) $A(q)A(q^2)$
- (12) $B(q)^2A(q^2)$
- (13) $B(q^2)A(q^2)$
- (14) $A(q^2)^2$
- (15) $A(q)/(1-q^2)$
- (16) $A(q)/(1-2q)$
- (17) $A(q)/(1-2q^2)$
- (18) $A(q)/(1-q)^2$
- (19) $A(q)/(1-2q^2)^2$
- (20) $A'(q)/(1-q)$
- (21) $(1+q)^kA(q)$
- (22) $A(q)/(1-q)$
- (23) $(1+q+q^2)A(q)$
- (24) $\frac{1-q^k}{1-q}A(q)$
- (25) $\frac{1}{2}(A(q) + A(-q))$
- (26) $\frac{1}{2}(A(q) - A(-q))$
- (27) $A(q) + qA'(q)$
- (28) $A(q) + 3qA'(q) + q^2A''(q)$
- (29) $A(q) + 7qA'(q) + 6q^2A''(q) + q^3A'''(q)$
- (30) $A'(q) + qA''(q)$
- (31) $A(q)^3$
- (32) $A(q)^4$
- (33) $1/(1-qA(q))$
- (34) $A(qB(q))$