FOURTH UNEXAM ASSIGNMENT

In this exercise I will ask you to prove an identity similar to Theorem 13-7 in the book. We will prove this identity using only basic principles of generating functions. Here is the idea: write down a generating function for the set of objects in the exercise as a product of generating functions for the parts of the partitions (since each of the sets of partitions below can be decomposed into a tuple consisting of the parts). This is the left hand side of the identity you are trying to prove.

Next each of the sets of partitions can also be written as a sum of disjoint subsets of partitions partitioned by the size of their largest Durfee square (or the largest first column, or the largest staircase, whichever you choose make it clear how you are breaking up the partitions). This might be an infinite disjoint union, but for partitions of size n you only need to calculate a finite number of terms so the expression will make sense.

A completed assignment also takes into account the parameter z which keeps track of the length of the permutation.

(1) Odd partitions.
(2) Partitions with parts of size 1 or 4 mod 5.
(3) Even partitions
(4) Odd strict partitions
(5) Strict partitions with parts of size 1 or 4 mod 5.
(6) Even strict partitions
(7) Odd partitions with at most 3 parts of any one size
(8) Partitions with parts of size 1 or 4 mod 5 and with at most 3 parts of any one size
(9) Even partitions with at most 3 parts of any one size
(10) Odd partitions with a multiple of 3 parts of any one size
(11) Partitions with a multiple of 3 parts of any one size
(12) Even partitions a multiple of 3 parts of any one size
(13) Odd partitions with at most k parts of size k (e.g. there are 0,1,2 or 3 parts of size 3)
(14) Partitions with at most k parts of size k
(15) Even partitions at most k parts of size k