UNEXAM #3

ASSIGNED: MARCH 3, 2016 , DUE: MARCH 17, 2016

Remember that the important aspect of this assignment is not the answer, but the solution. Please justify the following *using generating functions*. For the first problem you may want to use a computer computation but your argument may appeal to a calculation by computer (the coefficient of x^{98} in the generating function ... is 1712).

For the second question if you want to compare your answer to mine you will likely need a computer but this problem is interesting because the answer gives you very little information about how the algebraic equation was derived.

Please provide me with a single, clear, short solution which includes all details. Each of these problems boils down to essentially one thing: organize the question in a way that you can apply the multiplication or addition principle of generating functions. When you explain this you should tell me
(1) how you are organizing 'what you want' in a way you can apply basic principles of generating functions
(2) how to write down the generating function for each piece of the problem
(3) how to put these generating functions together using the addition and multiplication principle, and
(4) in the first problem how to find the numerical answer to the question (probably you will go ask a computer but I would like you to tell me what you asked the computer to determine for you).

Do not bother to try to explain algebra to me. I would prefer if you leave off any serious algebra from your explanation. That is not the point of this exercise.

Please, please, please *do not* try to make your answer for the second question look like mine, but you can use my answer to check to see if yours is right. Don't include that check in your solution! I want your argument to convince me your answer is right.

(1) At the beginning of the day, the cash register has 20 loonies, 20 toonies, 10 fives, 10 tens, 10 twenties and 4 fifties and 2 hundred dollar bills. A customer comes in and buys a \$2.00 cup of coffee with a hundred dollar bill. How many different ways are there of making change for the \$98 owed?

Answer: 1712

(2) What is the generating function for the number of solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = n$$

are there where each of the x_i are non-negative integers and $x_1 + x_2 + x_3 \equiv 1 \ (mod\ 2)$ and $x_4 \neq x_5$? Hint: The condition of $x_4 \neq x_5$ is the opposite of $x_4 = x_5$. Subtract the generating function for solutions with $x_4 = x_5$ from the generating function for all solutions.

Answer:

$$\frac{6x^2 + 2x^4}{1 - x - 4x^2 + 4x^3 + 6x^4 - 6x^5 - 4x^6 + 4x^7 + x^8 - x^9}$$

Bonus: Give me a formula for the coefficient of the coefficient of x^n in the generating function