

PARTITION GENERATING FUNCTIONS

A Durfee square is the largest square which can be fit inside of a partition.

Apply the addition or the multiplication principle of generating functions to give the generating function for the following sequences of numbers.

- (1) the number of partitions of n with parts that are multiples of 3.
- (2) the number of partitions of n with only even parts and all parts not occurring more than twice.
- (3) the number of partitions of n with distinct parts which are all congruent to 1 or 4 ($\text{mod } 5$).
- (4) the number of partitions of n with parts of size 1, 2 or 3 occurring at most 8 times each.
- (5) the number of partitions of n with at most 8 parts of any given size.
- (6) the number of partitions of n with even parts and 0, 1 or 2 parts of any given size.
- (7) the number of partitions of n with a Durfee square of odd size.
- (8) the number of partitions of n with distinct parts and no parts are equivalent to 3 ($\text{mod } 5$).
- (9) the number of partitions of n with less than or equal to k parts and each part occurs at most 4 times.
- (10) the number of partitions of n with with odd parts and a part will either occur 0 or an odd number of times.
- (11) the number of partitions of n into parts congruent to 2 or 3 ($\text{mod } 5$).
- (12) the number of partitions of n with length less than or equal to 5 and a Durfee square of size 1, 3 or 5.
- (13) the number of partitions of n with a Durfee square of odd size and all parts even.
- (14) the number of partitions of n with Durfee square equal to 3 and all odd parts.
- (15) the number of partitions of n with length exactly equal to 4.
- (16) the number of partitions of n with less than or equal to k parts and each part occurs an even number of times.
- (17) the number of partitions of n with no parts that are equivalent to 3 ($\text{mod } 5$).
- (18) the number of partitions of n with a Durfee square of square of size 1, 2 or 3.
- (19) the number of partitions of n with Durfee square equal to 4 and all distinct parts.
- (20) the number of partitions of n with distinct parts and all parts are equivalent to 3 ($\text{mod } 5$).
- (21) the number of partitions of n with length exactly equal to 5 and all parts distinct.
- (22) the number of partitions of n with even parts and all parts occurring an even number of times.
- (23) the number of partitions of n with odd parts and all parts occurring an even number of times.
- (24) the number of partitions of n with a Durfee square of size 3 and the length of the partition less than or equal to 4.
- (25) the number of partitions of n with distinct parts, a Durfee square of size 3 and length equal to 3.
- (26) the number of partitions of n with less than or 4 parts and all parts distinct.
- (27) the number of partitions of n with even parts occurring an even number of times and odd parts occurring any number of times.
- (28) the number of partitions of n with Durfee square equal to 3 and all even parts.
- (29) the number of partitions of n with length less than or equal to 3 and all parts distinct.
- (30) the number of partitions of n with odd parts and each part occurs at most 4 times.
- (31) the number of partitions of n with length exactly equal to 4 and all parts distinct.
- (32) the number of partitions of n with a Durfee square of even size and all parts even.