

Let P = plain text message
C = ciphertext
S = signed message

(e_A, N_A) = Alice's public key

(e_B, N_B) = Bob's public key

(d_A, N_A) = Alice's private key

(d_B, N_B) = Bob's private key

Alice wants to send Bob a message. She first performs the following calculations:

$C \equiv P^{e_B} \pmod{N_B}$ **Encryption of P with Bob's public key.**

$S \equiv C^{d_A} \pmod{N_A}$ **Creates S since only Alice knows her private key (Digital S)**

Bob gets the above message (S) and has to do the following:

$$S^{e_A} \equiv \left(\left[C^{d_A} \right] \right)^{e_A} \pmod{N_A}$$

$S^{e_A} \equiv C \pmod{N_A}$ **applies Alice's public key in order to verify signature.**

So, $S^{e_A} \equiv P^{e_B} \pmod{N_B}$ **Substitutes P for C from equation directly above.**

$$\Rightarrow C \equiv P^{e_B} \pmod{N_B}$$

$\Rightarrow C^{d_B} \equiv P^{e_B \times d_B} \pmod{N_B} \equiv P \pmod{N_B}$ **Retrieves plain text message from Alice using his own private key.**