

UNEXAM #3

ASSIGNED: FEBRUARY 21, 2008, DUE: MARCH 6, 2008

Remember that the important aspect of this assignment is not the answer, but the solution. Please justify the following *using generating functions*. For the first problem you may want to use a computer computation and for the second if you want to compare your answer to mine you will likely need a computer. Note that the first problem is also equivalent to a problem in number theory since it is equivalent to the number of solutions to the equation $x_1 + 3x_2 + x_3 = n$ with $x_i \geq 0$ and $x_3 \leq x_2$.

I will be grading you on a 5 point scale again. Please provide me with a single, clear, short solution which includes all details. Each of these problems boils down to essentially one thing: organize the question in a way that you can apply the multiplication or addition principle of generating functions. When you explain this you should tell me

- (1) how you are organizing ‘what you want’ in a way you can apply basic principles of generating functions
- (2) how to write down the generating function for each piece of the problem
- (3) how to put these generating functions together using the addition and multiplication principle, and
- (4) in the first problem how to find the numerical answer to the question (probably you will go ask a computer but I would like you to tell me what you asked the computer to determine for you).

Do not bother to try to explain algebra to me (in fact, I would prefer if you leave off any serious algebra from your explanation). That is not the point of this exercise.

Please, please, please *do not* try to make your answer for the second question look like mine.

- (1) In a game of calvinball, Calvin tromps on Hobbs and scores 100 points to Hobbs’ 0 (of course the rules were written in Calvin’s favor). Today’s rules for calvinball say that a player can score one point for hitting the other player in the head, 3 points for a goal but Calvin gets an extra point if the ball hits the tree behind the goal afterwards. How many different combinations of head hits, goals and extra points could have given the score of 100?

Answer: 451

- (2) What is the generating function for the number of solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + 2x_5 = n$$

with $x_i \geq 0$ and where $x_1 + x_2 + x_3$ is odd and less than or equal to 10, and $x_4 \neq x_5$?

Hint: The condition of $x_4 \neq x_5$ is the opposite of $x_4 = x_5$. Subtract the generating function for solutions with $x_4 = x_5$ from the generating function for all solutions.

Answer:

$$\frac{3q^2 + 6q^3 + q^4 + 2q^5 - 78q^{12} - 156q^{13} + 129q^{14} + 258q^{15} - 55q^{16} - 110q^{17}}{(1 - q^2)^4(1 - q^3)}$$