

WORKSHEET I : SEQUENCES AND SETS OF OBJECTS

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Say that a_0, a_1, a_2, \dots is a sequence of non-negative integers where a_n represents the number of “widgets of size n .” Assume similarly that b_n represents the number of “doodles of size n .” Below are a list of algebraic expressions labeled 1 through 19 and another list of combinatorial descriptions labeled (a) through (p). Match each one of the algebraic expressions with the combinatorial description such that the number of elements in the combinatorial description is equal to the expression. Here is the tough part: three of the equations do not have a combinatorial description. Write one for each of those.

- (1) $a_n + b_n$
- (2) $a_{n-1} + a_n$
- (3) $a_n + b_m$
- (4) $\binom{n}{k} a_n$
- (5) $\binom{n}{0} a_0 + \binom{n}{1} a_1 + \dots + \binom{n}{n-1} a_{n-1} + \binom{n}{n} a_n$
- (6) $a_0 + a_1 + a_2 + \dots + a_n$
- (7) $a_1 + 2a_2 + 3a_3 + \dots + na_n$
- (8) $a_n b_n$
- (9) $(a_0 + a_1 + \dots + a_n)(b_0 + b_1 + \dots + b_n)$
- (10) a_n^2
- (11) $a_1 a_2 \dots a_{n-1} a_n$
- (12) $a_0 b_n + a_1 b_{n-1} + \dots + a_n b_0$
- (13) $a_0 b_1 + a_1 b_2 + \dots + a_{n-1} b_n$
- (14) $a_0 b_0 + a_1 b_1 + a_2 b_2 + \dots + a_n b_n$
- (15) $\binom{n}{0} a_n b_0 + \binom{n}{1} a_{n-1} b_1 + \dots + \binom{n}{n-1} a_1 b_{n-1} + \binom{n}{n} a_0 b_n$
- (16) na_n
- (17) $a_n + a_{n-2} + \dots + a_{n \bmod 2}$
- (18) $na_0 + (n-1)a_1 + \dots + a_{n-1}$
- (19) $a_0 + a_2 + \dots + a_{2n}$

- (a) A pair consisting of a widget and a doodle such that the doodle is of size less than or equal to n and it is one size larger than the widget.
- (b) Sequences of length n where the k^{th} element of the sequence is a widget of size k .
- (c) A pair consisting of one widget and one doodle, both of the same size and each of size less than or equal to n .
- (d) A pair whose first element is a widget of size n and whose second element is a doodle of the same size.
- (e) A pair consisting of a subset of the numbers 1 through n and a widget which is the same size as the subset.
- (f) A pair consisting of a widget and a doodle, each one is of size less than or equal to n .
- (g) The number of widgets such that the size of the widget plus n is even.
- (h) A pair which is either a widget of size n and the number 1 or it is a doodle of size n and the number 2.
- (i) A pair whose first element is a subset of the integers 1 through n of size k and whose second element is a widget of size n .
- (j) A pair consisting of two widgets both of the same size and both of size n .
- (k) A pair consisting of a positive integer k and a widget such that the size of the widget plus the integer is less than or equal to n .
- (l) A widget of even size and of size less than or equal to $2n$.
- (m) A widget that is either of size n or $n-1$.
- (n) A pair consisting of a widget and a doodle such that the size of the widget plus the size of the doodle is n .
- (o) A triple consisting of a widget and a doodle and a subset of the integers 1 through n such that the widget and the subset are of the same size and the size of the widget plus the size of the doodle must be n .
- (p) A pair consisting of a widget whose size is less than or equal to n and an integer between 1 and the size of the widget.