Exercise 4 - Math 6121

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Result 1. *For all $g_1, g_2 \in G$, either $g_1H = g_2H$ or $g_1H \cap g_2H = \emptyset$*

Proof. Let z be some element of $g_1H \cap g_2H$. Then $z = g_1a$ for some $a \in H$, and $z = g_2b$ for some $b \in H$. If h is any element of H then $ah \in H$ and $a^{-1}h \in H$, since H is a subgroup of G. But $zh = g_1(ah)$ and $g_1h = z(a^{-1}h)$ for all $h \in H$. Therefore $zH \subset g_1H$ and $g_1H \subset zH$, and thus $g_1H = zH$. Similarly $g_2H = zH$, and thus $g_1H = g_2H$, as required. $\square$